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A novel theoretical proposition on the field emission of electrons under the action of centrifugal force

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Abstract: In this article we have suggested a novel idea of electron emission process from the surface of a metallic hollow sphere under the action of centrifugal force. The surface is assumed to be of a single layer metallic atoms. Therefore all the electrons are at the same radial distance from the centre of the sphere. The metallic sphere is assumed to be rotating about one of its symmetry axes with a uniform angular velocity. It may therefore be treated as a non-inertial frame. We have started with classical non-relativistic form of Hamiltonian for the electrons in such a uniformly rotating non-inertial frame and later make some quantum correction. In this theoretical study the conduction electrons in the metallic surface are assumed to be free classical gas. It has been noticed that in the Hamiltonian the single particle centrifugal potential, which is negative in nature is minimum at the equator. As a consequence free electrons will be accumulated inside the centrifugal potential well. We have proposed a mechanism to get electron emission from the equatorial zone under the action of centrifugal force. In the quantum picture it is of course tunneling from the potential well.

Keywords: Electrons; Field emission; Centrifugal force

1. Introduction

In the present article we have proposed a new mechanism of field emission of electrons (see [1–3] for the conventional model and also see [4,5] for field emission associated with magnetar). We have considered a hollow metallic sphere of extremely thin surface width, almost like a single layer of atoms. Therefore all the electrons are at a constant radial distance, say R from centre of the sphere. Then we can say that exactly like the celestial sphere, used in theoretical astronomy, the position of an electron is assigned by θ and ϕ the polar and the azimuthal coordinates. It has further been assumed that the free electrons on the metallic surface behave like a classical gas. The hollow sphere has a spinning motion about one of the symmetry axes. Since it is a metallic sphere, there are a lot of free electrons. These delocalized electrons can have mass motion during the rotation of the sphere. Whereas, the positively charged ions are rigidly attached in the spherical shape ionic lattice. Therefore they are at rest. The flow of electrons will be towards the negative centrifugal potential near the equator. The depth of the potential increases with the increase in angular velocity of the rotating sphere. The free electrons from rest of the places will be accumulated at the equator. It has been observed that in the steady state the free electrons will be accumulated near the equatorial zone within the angular width $\theta = 69^0$ to $\theta = 120^0$. This is also the range of the potential and is independent of the angular velocity of the sphere. Classically, outside this region is totally void. Not a single electron can be there in the void zone [7].

In this article, our investigation is purely theoretical in nature. We strongly believe that this theoretical proposition can be verified experimentally. It should further be noted that to the best of our knowledge, this problem has not been reported earlier.

We have arranged the article in the following manner: In the next section we have developed the basic formalism and in Sec. 3, a mechanism for the emission of electrons under the action of centrifugal force has been introduced and finally in Sec. 4, we have given the conclusion and discussed the importance of quantum correction.

2. Basic formalism

It is well known that a rotating sphere is equivalent to a non-inertial frame [8]. In the classical form of Hamiltonian for an electron, the potential has a minimum near the equator. The electrons should therefore be accumulated near the equatorial zone. To get some quantitative estimate, we follow Landau and Lifshitz [8]. We can write down the Lagrangian of a free electron in the following form:

$$L = \frac{1}{2}mv^2 + m\mathbf{v}.(\mathbf{\Omega}\times\mathbf{r}) + \frac{1}{2}(\mathbf{\Omega}\times\mathbf{r})^2 - U(\mathbf{r}), \tag{1}$$

where m and $\mathbf{v}$ are respectively the mass and the velocity of the electron, $\mathbf{\Omega}$ is the uniform angular velocity of the sphere and $U(\mathbf{r})$ is some background potential experienced by the electrons. Since the surface thickness of the hollow sphere is assumed to be extremely thin, almost like single atomic layer, the radial distances of all the free electrons from the centre are same. Hence we can write $\mid \mathbf{r} \mid = R$, the radius of the sphere. The background potential $U(R)$ is therefore a constant.

We have further assumed that the electron distribution is symmetric with respect to the azimuthal coordinate ϕ. Then the surface distribution of electrons depend only on the polar coordinate θ. Hence following the standard definition, the single particle Hamiltonian is given by

$$H = \mathbf{p}.\mathbf{v}(\mathbf{p}) - L. \tag{2}$$

Then with some little algebra it can very easily be shown that the Hamiltonian for an electron is given by

$$H = \frac{p^2}{2m} - \frac{1}{2}m\Omega^2R^2\sin^2\theta + U(R), \tag{3}$$

which may be re-expressed as

$$H = \frac{p^2}{2m} + V(\theta) + U(R), \tag{4}$$

where $V(\theta)$ is the well known centrifugal potential. Since only the free electrons can have mass motion and the metallic ions are immovable because they are tightly fixed in the ionic lattice by some rigidity force and also they are quite massive, we can write down the Poison's equation satisfied by the electron in the following form:

$$\nabla^2 V(\mathbf{r}) = 4\pi mGn_e. \tag{5}$$

Here $V(\mathbf{r})$ is equivalent to a gravitational potential arising from the centrifugal force experienced by the electrons on the rotating surface. Since the surface thickness is infinitesimally thin, the radius vector is a constant and from the rotational symmetry, the centrifugal potential is independent of the azimuthal coordinate. Then we can re-write the above equation in the form:

$$\frac{1}{R^2}\frac{1}{\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V(\theta)}{\partial\theta}\right) = 4\pi Gmn_e. \tag{6}$$

This is the relevant part of Poisson's equation. Substituting for $V(\theta)$, we have after some little algebra

$$4\pi Gmn_e = m\Omega^2(1-3\cos^2\theta) = m\Omega^2 x. \tag{7}$$

Since the left hand side is positive definite, the quantity $x = 1-3\cos^2\theta$ must also be ≥ 0 and this gives $|\cos\theta| \leq 1/3^{1/2}$. Therefore within the range of θ between 60^0 and 120^0, the values of n_e are non-zero, with maximum at $\theta = 90^0$. Beyond this range of θ, the electron density n_e becomes negative, which is unphysical. In Fig. 1 we have shown the variation of the quantity x with θ, where θ is from 0^0 to 180^0. It is quite obvious that x is non-zero positive definite for $60^0 \leq \theta \leq 120^0$. The result is independent of the azimuthal angle ϕ. The peak at $\theta = 90^0$ is because of the minimum. In Fig. 2 we have plotted the variation of $x(\theta,\phi)$ in three-dimension. Since the quantity $x(\theta,\phi)$ is independent of ϕ coordinate, we got a closely spaced repetition of the curve shown in Fig. 1.

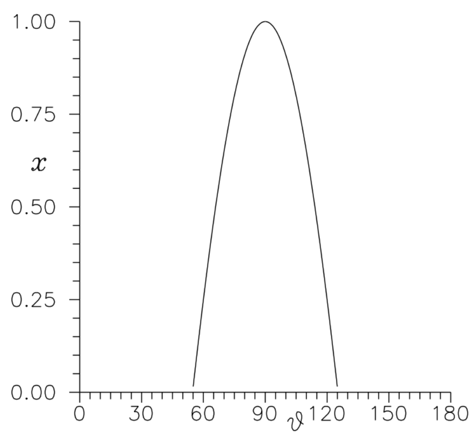

Fig. 1 Variation of x (introduced in the text) with θ

3. A possible new mechanism of electron emission

In this section we have introduced a new mechanism of field electron emission from the surface of the rotating metallic hollow sphere under the action of centrifugal force. To get electron emission, we propose the following changes near the equator. A large number of extremely thin needles with very sharp edges and made up of the same conducting material are put along the equator. These needles are projected radially outward with erected sharp edges. Another addition is a metallic belt along

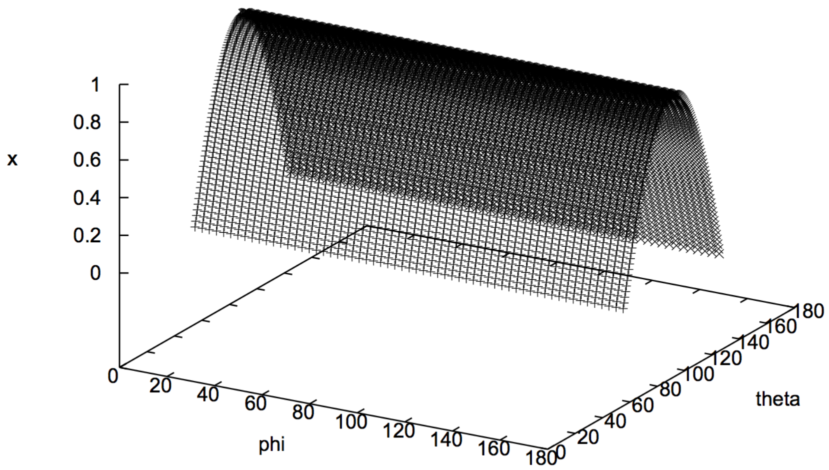

Fig. 2 Variation of x (introduced in the text) with θ and ϕ

the equator, but slightly above the sharp edges of the needles. It is like the zodiac belt of the celestial sphere, but not on the surface of the sphere at the equator. The angular width of the belt is less than 60^0. The negative potential region near the equatorial zone is now connected with the negative pole of a battery using silver-graphite brush connector. The belt is connected with the positive pole of the same battery. In the rotating condition, the circuit is completed along with a current measuring device. Since the needles are metallic, electrons will travel towards the edges and because of the sharp nature of the edges, following the mechanism of lightning arrester, electrons which are repelling each other at the sharp edges will go to the space and will move towards the positively charged metallic belt, which behaves like a positively charged cloud. As a consequence the circuit will show the flow of electric current. The current measuring device will show zero current when the sphere is at rest. As soon as it starts rotating some current will be seen in the measuring instrument. The intensity of current may not be quite high, but will be finite in magnitude (may be $\sim \mu$ amp) and measurable.

In this classical model, strength of field current is independent of the magnitude of angular velocity of the sphere. This can not be not be correct. But it is the limitation of the classical idea. In the conclusion we have shown that the strength of current will increase with the increase in $\mid \mathbf{\Omega} \mid$.

4. Conclusion

In this article we have proposed a mechanism of electron field emission. If it is found to be correct, then it will be a new mechanism of electron emission.

Finally we would like to mention, that from the classical point of view the strength of field current is independent of the magnitude of angular velocity of the sphere. This is not correct. The strength of current should increase with the rotational speed of the sphere. Of course this physical phenomenon can only be explained by quantum mechanics. For the proper explanation, one has to incorporate quantum correction. From Eq. (4) it is obvious that the depth of potential well is $\propto \Omega^2$. Now it is well known that the number of energy levels will increase with the depth of quantum mechanical potential well. Therefore when Ω is small enough, the potential well will be shallow, which in turn gives very few energy levels and as a consequence the number of electrons at the equatorial region will be low enough. The strength of field current may therefore be immeasurable for low Ω.

Because of the increase in the number of energy levels inside the potential well, which also gives enormous number of electrons, the strength of field current will increase with the increase in angular velocity of the sphere. With the increase in the magnitude of angular velocity, the continuum to bound transition probability will also increase. In the extreme case, when the angular velocity is large enough, the number of energy levels inside the potential well be so large and so closely spaced that they are almost like continuum and as a result in such situation, the transition will be like continuum to continuum. We expect a saturation of field current with the increase in angular velocity.

Therefore our final remark is that, the mechanism of field emission under the action of centrifugal force has not been discussed before.

References

1. R.H. Fowler and L. Nordheim, *Proc. Roy. Soc. (London)* **119** (1928) 173.
2. R.G. Forbes and J.H.B. Deane, *Proc. Roy. Soc. (London)* **463** (2007) 2907.
3. S.-D. Liang and L. Chen, *Phys. Rev. Lett.* **101** (2008) 027602.
4. A. Ghosh and S. Chakrabarty, *Eur. Phys. J. A* **47** (2011) 56.
5. A. Ghosh and S. Chakrabarty, *Mon. Not. R. Astron. Soc.* **425** (2012) 239.
6. A. Ghosh and S. Chakrabarty, *Astrophys. Astronomy* **32** (2011) 377.
7. S. Das, S. Ghosh and S. Chakrabarty, *Mod. Phys. Lett.* **32** (2017) 1750180.
8. L.D. Landau and E.M. Lifshitz, *Classical Mechanics* (Butterworth-Heimenann, Oxford, 1975).