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Study of physical parameters of the cosmological models of the universe in plane symmetric space-time in modified gravity

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Abstract: In this paper, we have intended to study the dynamics of the anisotropic universe in modified gravity using the functional f(R,T) in the form $f(R,T) = \lambda R + \lambda T$. The cosmological models are constructed using the volumetric exponential expansion in Bianchi type VI_h (BVI_h) universe for three different values of h = -1, 0, 1. The physical behaviors of the models are also studied.

Keywords: Modified Gravity: Perfect Fluid: Deceleration Parameter

1. Introduction

Since the beginning, there has been many significant accomplishments in the field of astrophysics and space science. Among all of them one of the most substantial achievement is Albert Einstein's general theory of relativity. This theory explains, warping of space-time as being the logic behind the observed gravitational phenomena and gives a complete insight of gravity as a geometric property of space-time. Thus relativity theory has crucial implications in present day astrophysics. Contrary to all, Einstein's general theory of relativity fails to explain the accelerated expansion of the universe. This swift extension of the universe is indicated by the recent cosmological data, which also determines that this accelerated expansion of the universe is due to some energy matter with negative pressure. Cosmologists have termed this energy matter as Dark Energy (DE) whose origin is still a suspense. DE has posed an elementary objection to all the gravitational theories. Introduction of "The Cosmological Constant" by Einstein which gives the energy density value of the vacuum space was a step towards explaining the cosmic acceleration but it encountered frequent problems due to considerable disparity between theory and observations [1]. Thus, in order to account for the cosmic acceleration General Relativity can be altered in contrasting ways. One of the ways which has gained enough praise is by modifying the underlying geometry. This is achieved by replacing the Einstein-Hilbert action through a random function. f(R) gravity (function of Ricci scalar R), f(T) gravity (function of scalar torsion T), f(G) gravity (function of G) and f(R,T) gravity (combined function of R and T) are among the few functions which are used to modify the Einstein-Hilbert action. It is considered that f(R) gravity is the most satisfactory function to realize the cosmic acceleration.

Among the several theoretical models proposed in regard of the dark energy and cosmic acceleration, a few are quintessence (Sahni and Starobinsky [2]; Padmanabhan [3]), cosmological constant (Weinberg [4]; Peebles and Ratra [5]), tachyon field (Padmanabhan [6]; Padmanabhan and choudhury [7]), quintom (Feng et al. [8];

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Guo et al. [9]), Chaplygin gas (Kamenshchik et al. [10]; Bento et al. [11]), holographic models (Wang et al. [12]; Setare and Shafei [13], Setare [14]; Hu and Ling [15]; Kim et al. [16]), phantom energy (Caldwell [17]; Nojiri and Odintosov [18]), k-essence, f-essence etc. On the basis of verification of the above theories, modified gravity has been successful in describing the late accelerated expansion of the universe. Thus, modified gravity nowadays is a topic of great interest. Out of all prospective variants of modified gravity, the f(R,T) gravity theory proposed by Harko et al. [19] is one of the most fascinating theories. In this theory they have generalized the basic f(R) theory by taking the gravitational Lagrangian as a random function of Ricci scalar R and trace of the stress-energy tensor T.

Inspired by the above argument and investigations in modified theories of gravity, in this paper, we offer to study a plane, static and symmetric space time in f(R,T) gravity by considering various functions of f(R,T) as proposed in [19] This paper has been organized as follows: In section 1, introductory discussion on modified gravity was made. In section 2, the field equations of f(R,T) gravity in a static, plane symmetric space time has been derived. Section 3 formulates the field equations for a plane, static and symmetric space-time in f(R,T) gravity considering various cases of f(R,T) i.e. (i) f(R,T)=R+2f(T) (ii) $f(R,T)=f_1(R)+f_2(T)$ (iii) $f(R,T)=f_1(R)+f_2(R)f_3(T)$ and giving the solutions for all the three cases. In section 4 physical and geometrical parameters of the models are defined and discussed along with their graphical plots. Finally conclusions are summarized in the last Section 5.

2. f(R,T) Gravity Theory

In f(R,T) gravity, Hilbert-Einstien type variational principle yields the gravitational field equations. The f(R,T) modified gravity action is given by

$$S = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4 x + \int L_m \sqrt{-g} d^4 x, \tag{1}$$

where f(R,T) is an arbitrary function of Ricci scalar R, T being the trace of the stress-energy tensor (T_{ij}) of the matter and L_m is the matter Lagrangian density. The stress-energy tensor of matter is defined as

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}.$$
 (2)

We assumed here that the dependence of matter Lagrangian is on the metric tensor g_{ij} rather than its derivatives.

The trace of the energy tensor of matter is given by

$$T = g^{ij}T_{ij}. (3)$$

So in this case the stress-energy tensor of matter is

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial q^{ij}}. (4)$$

Varying the action S of the gravitation field with respect to the metric tensor components g^{ij} , the field equations of f(R,T) gravity are obtained as follows

$$f(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T)$$

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$$=8\Pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij}$$
 (5)

where

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta} \frac{\partial^2 L_m}{\partial a^{ij}\partial g^{\alpha\beta}}.$$
 (6)

Now here $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$, $\Box \equiv \nabla^i \nabla_i$ where ∇^i is the covariant derivative and T_{ij} is the standard matter energy-momentum tensor.

Contraction of Eq. (5) yields

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)(T+\theta)$$
(7)

where $\theta = \theta_i^i$. The above Eq. (7) gives a relation between the trace T of energy-momentum tensor and Ricci scalar R.

It can be seen that when $f(R,T) \equiv f(R)$, Eq. (5) yields the field equations of f(R) gravity.

Now using Eq. (6), we get the variation of stress-energy. As there is no unique definition of matter Lagrangian, the matter Lagrangian can be taken as $L_m = p$.

Now using the Lagrangian L_m , the stress-energy tensor of matter is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \tag{8}$$

where $u^i = (0,0,0,1)$ is the four velocity vector in the co-moving coordinate system such that $u^i u_i = 1$ and $u^i \nabla_j u_i = 0$, ρ and p are energy density and pressure of the fluid respectively.

Then using Eq. (6), we obtain the variation of Stress-energy of perfect fluid as

$$\theta_{ij} = -2T_{ij} - pg_{ij}. (9)$$

On the physical nature of the matter field, the field equations also depend through the tensor θ_{ij} .

Hence in the case of f(R,T) gravity depending on the nature of matter source. We obtain several theoretical models for different matter contributions for f(R,T) gravity. Harko et al (2011) gave three classes of model as

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T). \end{cases}$$
 (10)

In this paper we focus on all the three above mentioned cases. Firstly we focus on the first case, i.e., f(R,T) = R + 2f(T) where f(T) is an arbitrary function of Stress-Energy tensor of matter. Now from Eq. (5) we get the field equations of f(R,T) gravity as

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij},$$
(11)

where prime denotes differentiation with respect to the argument.

In perfect fluid the field equations become

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}.$$
 (12)

Then we focus on the second case, i.e., $f(R,T) = f_1(R) + f_2(T)$ where $f_1(R)$ is an arbitrary function of Ricci Scalar and $f_2(T)$ is an arbitrary function of Stress-Energy tensor of matter. For this case in perfect fluid the field equations develop into

$$f_1'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} = 8\pi T_{ij} + f_2'(T)T_{ij} + [f_2'(T)p + \frac{1}{2}f_2(T)]g_{ij}.$$
 (13)

Finally we focus on the third case, i.e., $f(R,T) = f_1(R) + f_2(R)f_3(T)$ where $f_1(R)$ and $f_2(R)$ is an arbitrary function of Stress-Energy tensor of matter. For this case in perfect fluid the field equations develop into

$$[f_1'(R) + f_2'(R)f_3(T)]R_{ij} - \frac{1}{2}f_1(R)g_{ij}$$

$$= 8\pi T_{ij} + f_2(R)f_3'(T)T_{ij} + f_2(R)[f_3'(T)p + \frac{1}{2}f_3(T)]g_{ij}.$$
(14)

3. Field Equations and its solutions

We have considered a static, plane and symmetric space-time of the form

$$ds^{2} = A^{2}(dt^{2} - dx^{2}) - B^{2}(dy^{2} + dz^{2}),$$
(15)

where A(t) and B(t) are the two anisotropic directions of the space and functions of cosmic time only.

These functions are not equal due to radial asymmetry. The matter tensor can be defined as

$$\theta_{ij} = -2T_{ij} - pg_{ij} = (\rho, -p, -p, -p).$$
 (16)

Now the formulation of equations and their solution for each of the three cases are shown.

3.1 Case I:

$$f(R,T) = R + 2f(T). \tag{17}$$

With specific function choice $f(T) = \lambda T$, where λ is a constant, the field equations are obtained as

$$\frac{2\ddot{B}}{A^2B} + \frac{\dot{B}^2}{A^2B^2} - \frac{2\dot{A}\dot{B}}{A^3B} = (8\pi + 3\lambda)p - \lambda\rho,\tag{18}$$

$$\frac{\ddot{B}}{A^2B} - \frac{\dot{A}^2}{A^4} + \frac{\ddot{A}}{A^3} = (8\pi + 3\lambda)p - \lambda\rho,\tag{19}$$

$$\frac{\ddot{B}}{A^2B} - \frac{\dot{A}^2}{A^4} + \frac{\ddot{A}}{A^3} = (8\pi + 3\lambda)p - \lambda\rho,\tag{20}$$

$$\frac{2\dot{A}\dot{B}}{A^{3}B} + \frac{\dot{B}^{2}}{A^{2}B^{2}} = -(8\pi + 3\lambda)\rho - \lambda p, \tag{21}$$

where an overhead dot represents differentiation with respect to cosmic time 't'.

As Eqs. (19) and (20) are same, therefore we have four unknowns namely A, B, ρ , ρ and three equations. Without loss of generality we take

$$A = B^m, (22)$$

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where m is an arbitrary constant.

On solving the above Eqs. (18) - (22) we get the solution as

$$A = (k_1 t + k_2)^{m/2}, (23)$$

$$B = (k_1 t + k_2)^{1/2}, \tag{24}$$

where k_1 and k_2 are arbitrary constants. p and ρ for this case are obtained as

$$p = \rho = \frac{-(2m+1)k_1^2}{8(4\pi+\lambda)(k_1t+k_2)^{m+2}}.$$
 (25)

3.2 Case II:

$$f(R,T) = f_1(R) + f_2(T). (26)$$

With specific function choice $f_1(R) = \lambda R$ and $f_2(T) = \lambda T$, where λ is a constant, the field equations are obtained as

$$\frac{2\ddot{B}}{A^2B} + \frac{\dot{B}^2}{A^2B^2} - \frac{2\dot{A}\dot{B}}{A^3B} = (\frac{8\pi + \lambda}{\lambda} + \frac{1}{2})p - \frac{\rho}{2},\tag{27}$$

$$\frac{\ddot{B}}{A^2B} - \frac{\dot{A}^2}{A^4} + \frac{\ddot{A}}{A^3} = (\frac{8\pi + \lambda}{\lambda} + \frac{1}{2})p - \frac{\rho}{2},\tag{28}$$

$$\frac{\ddot{B}}{A^2B} - \frac{\dot{A}^2}{A^4} + \frac{\ddot{A}}{A^3} = (\frac{8\pi + \lambda}{\lambda} + \frac{1}{2})p - \frac{\rho}{2},\tag{29}$$

$$\frac{2\dot{A}\dot{B}}{A^{3}B} + \frac{\dot{B}^{2}}{A^{2}B^{2}} = \frac{p}{2} - (\frac{8\pi + \lambda}{\lambda} + \frac{1}{2})\rho,\tag{30}$$

where an overhead dot represents differentiation with respect to cosmic time t'.

As Eqs. (28) and (29) are same, therefore we have four unknowns namely A, B, ρ , ρ and three equations. Without loss of generality we take

$$A = B^m, (31)$$

where 'm' is an arbitrary constant.

On solving the above Eqs. (27) - (31) we get the solution as

$$A = (k_1 t + k_2)^{m/2}, (32)$$

$$B = (k_1 t + k_2)^{1/2}, (33)$$

where k_1 and k_2 are arbitrary constants. For this case p and ρ are obtained as

$$p = \rho = \frac{-\lambda(2m+1)k_1^2}{4(8\pi+\lambda)(k_1t+k_2)^{m+2}}.$$
(34)

3.3 Case III:

$$f(R,T) = f_1(R) + f_2(R)f_3(T). (35)$$

With specific function choice $f_1(R) = \lambda R$, $f_2(R) = \lambda R$ and $f_3(T) = \lambda T$, where λ is a constant, the field equations are obtained as

$$\frac{2\ddot{B}}{A^{2}B} + \frac{\dot{B}^{2}}{A^{2}B^{2}} - \frac{2\dot{A}\dot{B}}{A^{3}B} = \frac{8\pi p}{\lambda + \lambda^{2}\rho - 3\lambda^{2}p},$$
(36)

$$\frac{\ddot{B}}{A^2B} - \frac{\dot{A}^2}{A^4} + \frac{\ddot{A}}{A^3} = \frac{8\pi p}{\lambda + \lambda^2 \rho - 3\lambda^2 p},\tag{37}$$

$$\frac{\ddot{B}}{A^2B} - \frac{\dot{A}^2}{A^4} + \frac{\ddot{A}}{A^3} = \frac{8\pi p}{\lambda + \lambda^2 \rho - 3\lambda^2 p},\tag{38}$$

$$(2\lambda^{2}\rho + 2\lambda^{2}p)\left[\frac{\dot{A}^{2}}{A^{2}} - \frac{\ddot{A}}{A} - \frac{2\ddot{B}}{B}\right] + (\lambda^{2}p - \lambda - 3\lambda^{2}\rho)\frac{\dot{B}^{2}}{B^{2}} + (6\lambda^{2}p - 2\lambda - 2\lambda^{2}\rho)\frac{\dot{A}\dot{B}}{AB} = 8\pi\rho A^{2},$$
(39)

where an overhead dot represents differentiation with respect to cosmic time it'.

As Eqs. (37) and (38) are same, therefore we have four unknowns namely A, B, ρ , ρ and three equations. Without loss of generality we take

$$A = B^m, (40)$$

where m is an arbitrary constant.

On solving the above Eqs. (36) - (40), we get the solution as

$$A = (k_1 t + k_2)^{m/2}, (41)$$

$$B = (k_1 t + k_2)^{1/2}, \tag{42}$$

where k_1 and k_2 are arbitrary constants. ρ and p for this case are obtained as

$$\rho = \frac{\left(\frac{m^2}{2} + \frac{m}{2} + \frac{1}{8}\right)\lambda^3 y^2 + (4\pi m + 2\pi)\lambda y}{\left(16\pi m + 2\pi\right)\lambda^2 y - \left(2m^2 + \frac{5m}{4} + \frac{1}{8}\right)\lambda^4 y^2 - 64\pi^2},\tag{43}$$

$$p = \left(\frac{8\pi - m\lambda^2 y}{m\lambda^2 y + \frac{\lambda^2 y}{4} + 8\pi}\right)\rho,\tag{44}$$

where

$$y = \frac{k_1^2}{(k_1 t + k_2)^{m+2}}. (45)$$

4. Physical and Geometrical Interpretations

In this section we define and study the physical and geometrical parameters of our model. All the physical and geometrical parameters discussed below are same for all the three cases. The volume scale factor V and the average scale factor R are obtained respectively as

$$V = \sqrt{(-g)} = A^2 B^2 = (k_1 t + k_2)^{m+1}, \tag{46}$$

$$R = V^{1/3} = (k_1 t + k_2)^{\frac{m+1}{3}}. (47)$$

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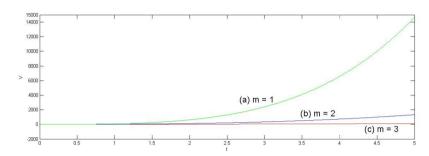


Fig. 1 V versus t for (a) m = 1, (b) m = 2 and (c) m = 3 where $k_1 = 2$, $k_2 = 1$.

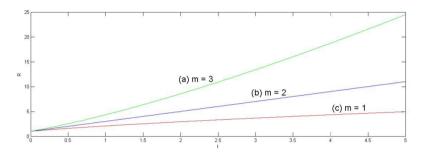


Fig. 2 R versus t for (a) m=1, (b) m=2 and (c) m=3 where $k_1=2,\,k_2=1.$

The model signifies that the spatial volume increases with increase in time and points out the expanding nature of the space-time. The nature of volume scale factor V and the average scale factor R can be inferred from Figs. 1 and 2 respectively for various values of constant M.

Analogous to this model the scalar expansion ' θ ' is given as

$$\theta = \frac{\dot{V}}{V} = \frac{(m+1)k_1}{k_1t + k_2}.\tag{48}$$

The Hubble parameter H is given as

$$H = \frac{\theta}{3} = \frac{(m+1)k_1}{3(k_1t + k_2)} \tag{49}$$

which comes out to be a function of t'.

Measure of the cosmic accelerated expansion of the universe is defined by the deceleration parameter 'q'. Value of the deceleration parameter governs the nature of the space-time. Positive value indicates decelerating model whereas negative value indicates accelerating model. Now, we define the decelerating parameter 'q' as

$$q = \frac{-R\ddot{R}}{\dot{R}^2}. (50)$$

For our model the value of q is obtained as

$$q = -(\frac{m-2}{m+1}). (51)$$

The deceleration parameter attains the following values depending on different values of \boldsymbol{m}

$$q = \begin{cases} > 0 & 0 < m < 2 \\ = 0 & m = 2 \\ < 0 & m > 2. \end{cases}$$
 (52)

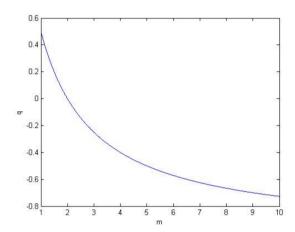


Fig. 3 q vs m where 1 <= m <= 10

Hence the model shows decelerating nature for 0 < m < 2, remains stagnant at m = 2 and accelerating nature for m > 2. Behavior of the deceleration parameter 'q' with constant 'm' is shown in Fig. 3.

The Shear scalar ' σ ' is defined as follows

$$\sigma^2 = \frac{1}{2} \left[\sum H_i^2 - \frac{\theta^2}{3} \right]. \tag{53}$$

For our model it comes out to be

$$\sigma^2 = \frac{(2 - m^2 - 8m)k_1^2}{24(k_1t + k_2)^2}. (54)$$

The Ricci scalar for all the models is found to be

$$R = (k_1 t + k_2)^{\frac{m+1}{3}}. (55)$$

As $t\to 0,\,R\to k_2^{\frac{m+1}{3}}$ and as $t\to \infty$, $R\to \infty.$ Therefore, it can be seen that the curvature of the space-time is increasing continuously with time and approaches infinity at infinite time. The trace of the stress-energy tensor 'T' for the three cases are respectively as follows

4.1.1 Case I:

$$T = \frac{(2m+1)k_1^2}{4(4\pi+\lambda)(k_1t+k_2)^{m+2}}.$$
 (56)

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4.1.2 Case II:

$$T = \frac{\lambda(2m+1)k_1^2}{2(8\pi+\lambda)(k_1t+k_2)^{m+2}}.$$
 (57)

4.1.3 Case III:

$$T = \left(\frac{4m\lambda^2 y + \frac{\lambda^2 y}{4} - 16\pi}{m\lambda^2 y + \frac{\lambda^2 y}{4} + 8\pi}\right)\rho,\tag{58}$$

where

$$\rho = \frac{\left(\frac{m^2}{2} + \frac{m}{2} + \frac{1}{8}\right)\lambda^3 y^2 + (4\pi m + 2\pi)\lambda y}{\left(16\pi m + 2\pi\right)\lambda^2 y - \left(2m^2 + \frac{5m}{4} + \frac{1}{8}\right)\lambda^4 y^2 - 64\pi^2}$$
(59)

and

$$y = \frac{k_1^2}{(k_1 t + k_2)^{m+2}}. (60)$$

Now, the function f(R,T) for all the three models are as follows

4.2.1 Case I:

$$f(R,T) = R + 2\lambda T = \left[(k_1 t + k_2)^{\frac{m+1}{3}} + \lambda \frac{(2m+1)k_1^2}{2(4\pi + \lambda)(k_1 t + k_2)^{m+2}} \right].$$
 (61)

4.2.2 Case II:

$$f(R,T) = \lambda(R+T) = \lambda[(k_1t + k_2)^{\frac{m+1}{3}} + \frac{\lambda(2m+1)k_1^2}{2(8\pi+\lambda)(k_1t + k_2)^{m+2}}].$$
 (62)

4.2.3 Case III:

$$f(R,T) = \lambda R(1+\lambda T) = \left[\lambda (k_1 t + k_2)^{\frac{m+1}{3}} \left(1 + \lambda \left(\frac{4m\lambda^2 y + \frac{\lambda^2 y}{4} - 16\pi}{m\lambda^2 y + \frac{\lambda^2 y}{4} + 8\pi}\right)\rho\right)\right], \quad (63)$$

where

$$\rho = \frac{\left(\frac{m^2}{2} + \frac{m}{2} + \frac{1}{8}\right)\lambda^3 y^2 + (4\pi m + 2\pi)\lambda y}{\left(16\pi m + 2\pi\right)\lambda^2 y - \left(2m^2 + \frac{5m}{4} + \frac{1}{8}\right)\lambda^4 y^2 - 64\pi^2}$$
(64)

and

$$y = \frac{k_1^2}{(k_1 t + k_2)^{m+2}}. (65)$$

The geometrical nature of the model is described by the state finder diagnostic pair $\{r,s\}$ which are defined as

$$r = \frac{\ddot{R}}{RH^3} \tag{66}$$

and

$$s = \frac{r-1}{3(q-\frac{1}{2})}. (67)$$

For all the three models discussed above the state finder diagnostic pair $\{r,s\}$ is found to be

$$r = \frac{(m-2)(m-5)}{(m+1)^2} \tag{68}$$

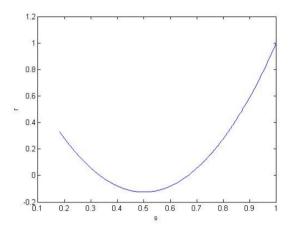


Fig. 4 r vs s where $1 \le m \le 10$

and

$$s = \frac{2}{(m+1)},\tag{69}$$

where m is an arbitrary constant. Fig. 4 depicts the plot of state finder diagnostic pair r with s.

5. Conclusions

The cosmological models of the universe has been framed in an anisotropic space-time with the three choices of f(R,T) gravity. The dynamical parameters are derived with an assumption between the scale factors. The physical parameters of the models are studied along-with the state finder pair. The models presented here provide a systematic mathematical derivation and the graphical representation shows the physical viability of the model. We conclude that further physical investigations can be performed to get more insight to its behavior.

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