Scientific Voyage
Vol. 2, No. 2,
Page 26-34 (2021)

SCIENTIFIC VOYAGE
ISSN: 2395-5546

GCECT
Publication
Apr. - Jun.

Dynamical and geometrical analysis of scale factor based solution in general relativity

Eesha Gadia[1] and Shruti Porwal[2]

Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad 500078, India

Abstract: In this paper, we have derived Einstein's field equations in an isotropic spacetime. The dynamical parameters are derived with assumed scale factors and analyzed. All the models are tested through geometrical diagnostic methods and luminosity distance. The stability analysis on the assumptions made are also performed. A detail derivation of all the dynamical parameters and other relevant parameters are done.

Keywords: General Relativity; Hubble Parameter; Dynamical Parameter; Geometrical Parameter

1. Introduction

In modern cosmology, the main theme of research is on the issue of late time cosmic acceleration phenomena. Supernovae type Ia provides strong evidence that the universe is currently undergoing an accelerated phase of expansion. Plethora of observations during the last two decades have confirmed the late time cosmic acceleration of the universe [1,2,3,4,5,6,7]. These observations have developed a curiosity among the cosmologists to explain this late time dynamics. In the purview of General Relativity (GR), it becomes difficult to explain this issue. Therefore, the idea of an unusual and exotic dark energy (DE) form with negative pressure has been surfaced. The most intriguing thing is that, so far we do not exactly know the nature and origin of this exotic energy source. The contribution of DE as compared to the baryonic matter and dark matter in providing an anti gravity effect to drive apart the universe for acceleration is the maximum. Recent Planck data estimates a lion share of 68.3 percent in favour of DE [8,9,10]. The late time cosmic dynamics and the consequent dark energy is understood through a dark energy equation of state parameter $\omega = \frac{p}{\rho}$, where p and ρ respectively denote the pressure and energy density of dark energy. There are approaches to study the dark energy models, (i) geometrically modifying the gravity or (ii) incorporating dark energy pressure and energy density in the matter part. We prefer the second approach here.

In this project, we have derived the field equations of Einstein's filed equation in FRW space-time and the energy momentum tensor is that of dark energy. In the field equations we have incorporated scale factors to get the background cosmology. In Sec. 2, we have derived the field equations. In Sec. 3, the solutions of the field equations using scale factors are obtained. The stability analysis and the luminosity distance are obtained for the models in Secs. 4 and 5 respectively. Concluding remarks are given in Sec. 6.

[1] E-mail: eesha.gadia123@gmail.com
[2] E-mail: shruti.porwal25@gmail.com

2. Spacetime and field equations

In order to construct the cosmological model of the universe, we have considered flat Friedmann-Robertson-Walker spacetime (FRW) in the form

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2 + dz^2), \tag{1}$$

where $a = a(t)$ be the average scale factor.

The spherical symmetric form of Eq. (1) can be written as

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2). \tag{2}$$

The Einstein's field equations can be expressed as

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = \kappa T_{ij}, \tag{3}$$

where G_{ij}, R_{ij}, R, g_{ij}, T_{ij} respectively represent the Einstein tensor, Ricci tensor, Ricci scalar, invariant and energy momentum tensor. The invariant g_{ij} can be represented in matrix form as

$$g_{ij} = \begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & a^2r^2 & 0 & 0 \\ 0 & 0 & a^2r^2\sin^2\theta & 0 \\ 0 & 0 & 0 & -1. \end{pmatrix} \tag{4}$$

The contravariant form of g^{ij} can be calculated as

$$g^{ij} = \frac{\text{cofactor of } g_{ij} \text{ in g}}{|g_{ij}|} \tag{5}$$

$$g_{ij} = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 & 0 \\ 0 & \frac{1}{a^2r^2} & 0 & 0 \\ 0 & 0 & \frac{1}{a^2r^2\sin^2\theta} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{6}$$

From the matrix, we can calculate the Christoffel symbols as

$$\Gamma^a_{bc} = \Gamma^a_{bc,k} = \frac{g^{ak}}{2}\left[\frac{\partial g_{bk}}{\partial x^c} + \frac{\partial g_{ck}}{\partial x^b} - \frac{\partial_{bc}}{\partial x^k}\right]. \tag{7}$$

The surviving Christoffel symbols are:

$\Gamma^1_{14} = \frac{\dot{a}}{a}$, $\Gamma^1_{22} = -r$, $\Gamma^1_{33} = -r\sin^2\theta$, $\Gamma^2_{12} = \frac{1}{r}$,

$\Gamma^2_{24} = \frac{\dot{a}}{a}$, $\Gamma^2_{33} = -\sin\theta\cos\theta$, $\Gamma^3_{13} = \frac{1}{r}$, $\Gamma^3_{23} = \cot\theta$,

$\Gamma^3_{34} = \frac{\dot{a}}{a}$, $\Gamma^4_{11} = a\dot{a}$, $\Gamma^4_{22} = a\dot{a}r^2$, $\Gamma^4_{33} = a\dot{a}r^2\sin^2\theta$.

Now, the Ricci tensor R_{ij} can be calculated using the formula

$$R_{ij} = R^a_{ij,a} = \frac{\partial}{\partial x^j}\Gamma^b_{ib} - \frac{\partial}{\partial x^b}\Gamma^b_{ij} + \Gamma^a_{ib}\Gamma^b_{ja} - \Gamma^a_{ij}\Gamma^b_{ab}. \tag{8}$$

For the spacetime 2, the Ricci tensor can be calculated as

$$
\begin{aligned}
R_{11} &= -a^2\left(\frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2}\right),\\
R_{22} &= -a^2r^2\left(\frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2}\right),\\
R_{33} &= -a^2r^2\sin^2\theta\left(\frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2}\right),\\
R_{44} &= 3\frac{\ddot{a}}{a}.
\end{aligned} \tag{9}
$$

Now, the Ricci Scalar $R = g^{ij}R_{ij}$ can be calculated as

$$R = -6\left(\frac{\dot{a}^2}{a^2}+\frac{\ddot{a}}{a}\right). \tag{10}$$

The energy momentum tensor can be described in the form of perfect fluid as

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}, \tag{11}$$

where p and ρ respectively denote the pressure and energy density of the matter field. Now, the non-vanishing energy momentum tensor can be calculated as

$$
\begin{aligned}
T_{11} &= pa^2,\\
T_{22} &= pa^2r^2,\\
T_{33} &= pa^2r^2\sin^2\theta,\\
T_{44} &= \rho.
\end{aligned}
$$

Now, we can obtain the Einstein's field equations (3) in the following form

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2} = -\kappa p, \tag{12}$$

$$3\frac{\dot{a}^2}{a^2} = \kappa\rho. \tag{13}$$

3. Solution of the field Equations

In order to understand the background cosmology, we need to incorporate the scale factor to obtain the solution to the filed equations. We have studied the cosmological model by incorporating two types of scale factor such as (i) quasi linear scale factor, (ii) linear combinations of scale factor and (iii) bouncing scale factor.

3.1 Quasi linear scale factor

The quasi linear scale factor can be expressed in the form $a = e^{H_o t} - \frac{M^2t^2}{12}$, where H_0 is the present value of the Hubble parameter and M be a constant. Subsequently the Hubble parameter H can be derived as $H = \frac{\dot{a}}{a} = H_0 - \frac{M^2 t}{6}$.

Incorporating this form of quasi linear scale factor in Einstein's field equations (12) – (13), we can derive the pressure, energy density as

$$p=\frac{1}{\kappa}\left[\frac{\frac{1}{3}H_0M^2te^{H_0t}+\frac{M^2}{3}e^{H_0t}+\frac{M^2}{6}H_0^2t^2e^{H_0t}-\frac{M^4t^2}{18}-3H_0^2e^{2H_0t}}{(e^{H_0t}-\frac{M^2t^2}{12})^2}\right], \tag{14}$$

$$\rho=\frac{1}{\kappa}[3H^2]=\frac{3}{\kappa}\left[\frac{H_0e^{H_0t}-\frac{M^2t}{6}}{e^{H_0t}-\frac{M^2t^2}{12}}\right]^2=\frac{3}{\kappa}\left[\frac{H_0^2e^{2H_0t}+\frac{M^4t^2}{36}-\frac{H_0}{3}M^2te^{H_0t}}{e^{2H_0t}+\frac{M^4t^4}{144}-\frac{M^2}{6}t^2e^{H_0t}}\right] \tag{15}$$

Subsequently the equation of state (EoS) parameter $\omega=\frac{p}{\rho}$ can be calculated as

$$\omega=\frac{p}{\rho}=-\frac{2}{3}\left[\frac{H_0^2e^{2H_0t}-\frac{1}{12}H_0^2e^{H_0t}M^2t^2-\frac{1}{6}M^2e^{H_0t}+\frac{1}{72}M^4t^2}{(H_0e^{H_0t}-\frac{M^2t}{6})^2}+\frac{1}{3}\right]. \tag{16}$$

The Null Energy Condition (NEC), Strong Energy Condition (SEC) and Dominant Energy Condition (DEC) can be respectively calculated as,

$$\rho+p=\frac{M^4t^2+6M^2e^{H_ot}((H_ot-2)^2-2)}{36\kappa\left(e^{H_ot}-\frac{M^2t^2}{12}\right)^2}, \tag{17}$$

$$\rho+3p=\frac{6}{\kappa}\left[\frac{\frac{M^2}{6}-H_o^2e^{H_ot}}{e^{H_ot}-\frac{M^2t^2}{12}}\right], \tag{18}$$

$$\rho-p=\frac{2}{\kappa}\left[\frac{3H_o^2e^{2H_ot}-\frac{H_o^2e^{H_ot}M^2t^2}{12}-\frac{2}{3}H_oe^{H_ot}M^2t-\frac{1}{6}M^2e^{H_ot}+\frac{5}{72}M^4t^2}{(e^{H_ot}-\frac{M^2t^2}{12})^2}\right] \tag{19}$$

The deceleration parameter, scalar expansion and the spatial volume can be obtained as

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=\frac{-(H_0^2e^{2H_0t}-\frac{e^{H_0t}M^2}{6}-\frac{M^2H_0^2e^{H_0t}t^2}{12}+\frac{M^4t^2}{72})}{H_0^2e^{2H_0t}-\frac{2M^2tH_0e^{H_0t}}{6}+\frac{M^4t^2}{36}}, \tag{20}$$

$$\theta=3H=3\frac{\dot{a}}{a}=3\frac{\left[H_0e^{H_0t}-\frac{M^2t}{6}\right]}{\left[e^{H_0t}-\frac{M^2t^2}{12}\right]}, \tag{21}$$

$$V=a^3=\left[e^{H_ot}-\frac{M^2t^2}{12}\right]^3. \tag{22}$$

The geometrical diagnostic, state finder pair $(r,\ s)$ can be obtained as

$$r=\frac{\ddot{H}}{H^3}-(2+3q)=\frac{a^2\dddot{a}}{\dot{a}^3} \tag{23}$$
$$=\frac{H_0^3e^{3H_0t}+\frac{M^4H_0^3t^4e^{H_0t}}{144}-\frac{M^2H_0^3t^2e^{2H_0t}}{6}}{H_0^3e^{3H_0t}+\frac{M^4H_0t^2e^{H_0t}}{12}-\frac{3M^2tH_0^2e^{2H_0t}}{6}-\frac{M^6t^3}{216}},$$

$$s=\frac{r-1}{3(q-0.5)}=\frac{2\left[\dddot{a}a^2-\dot{a}^2\right]}{3\dot{a}\left[-2a\ddot{a}-\dot{a}^2\right]} \tag{24}$$
$$=\frac{2\left[\left(\frac{tH_0^2}{2}-\frac{t^2H_0}{6}\right)M^2e^{H_0t}+\left(H_0^3t^4-\frac{5H_0t^2}{12}\right)M^2e^{H_0t}+\frac{M^6t^3}{216}\right]}{\left[H_0^3e^{3H_0t}-\left(\frac{H_0^3t^2}{6}-\frac{H_0^2t}{6}+\frac{H_0}{3}\right)M^2e^{2H_0t}+\left(\frac{H_0^3t^2}{36}-\frac{H_0^2t}{18}+\frac{t}{18}\right)M^4e^{2H_0t}\right]}.$$

Another geometric diagnostic methods is the $Om(z)$ diagnostic that involves first derivative of the scale factor and therefore becomes easier to apply to distinguish between different dark energy models. The $Om(z)$ parameter can be defined as

$$Om(z) = \frac{E^2(z)-1}{(1+z)^3-1} = \frac{\dot{z}^2 - H_0(1+z)^2}{H_0^2(1+z)^2\left[(1+z)^3-1\right]} \tag{25}$$
$$= \frac{\left(e^{H_0t-\frac{M^2t^2}{12}}\right)\left(H_0e^{H_0t-\frac{M^2t}{6}}\right)^2 - H_0^2\left(e^{H_0t-\frac{M^2t^2}{12}}\right)}{H_0^2\left(1-e^{H_0t}-\frac{M^2t^2}{12}\right)},$$

where $E(z) = \frac{Hz}{H_0}$ is the dimensionless Hubble parameter and H_0 is the Hubble rate at present epoch. Also, the red shift can be expresses with respect to the scale factor as, $z = \frac{1}{a} - 1$. Now, the throat radius of Morris-Thorne wormhole can be obtained as

$$R = -\frac{\kappa a}{2c\dot{a}} = -\frac{\kappa(e^{H_0t} - \frac{M^2t^2}{12})}{2c(H_0e^{H_0t} - \frac{M^2t}{6})}. \tag{26}$$

It has been observed that when $t \to 0$, then $R \to -\frac{k}{2cH_0}$ whereas when $t \to \infty$, then $R \to -\frac{k}{2cH_0}$. $R = 0$ when $H_0t - 2lnt = ln\frac{M^2}{12}$.

3.2 Linear combination of exponential scale factor

Another important scale factor to study the background cosmology is the linear combination of exponential scale factor which can be described as $a(t) = \sigma e^{\lambda t} + \tau e^{-\lambda t}$. Unlike in exponential scale factor, this scale factor consisting of both positive and negative exponential factor. The significance of the scale factor is that when $\sigma = 0$, the scale factor reduces to negative exponential factor whereas for $\tau = 0$, it reduces to positive exponential function. The Hubble parameter H can be obtained as $H = \frac{\dot{a}}{a} = \frac{\sigma\lambda e^{\lambda t} - \tau\lambda e^{-\lambda t}}{\sigma e^{\lambda t} + \tau e^{-\lambda t}}$ and subsequently $\dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = \frac{4\sigma\tau\lambda^2}{(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2}$. Now from the Einstein's field equations in FRW space-time, the pressure, energy density and EoS parameter $\omega = \frac{p}{\rho}$ can be obtained as

$$p = -\frac{\lambda^2}{\kappa}\left[\frac{3\sigma^2e^{2\lambda t} + 3\tau^2e^{-2\lambda t} + 2\sigma\tau}{(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2}\right], \tag{27}$$

$$\rho = \frac{3\lambda^2}{\kappa}\left[\frac{\sigma e^{\lambda t} - \tau e^{-\lambda t}}{\sigma e^{\lambda t} + \tau e^{-\lambda t}}\right]^2, \tag{28}$$

$$\omega = -\frac{1}{3}\left[\frac{3\sigma^2e^{2\lambda t} + 3\tau^2e^{-2\lambda t} + 2\sigma\tau}{(\sigma e^{\lambda t} - \tau e^{-\lambda t})^2}\right]. \tag{29}$$

The deceleration parameter, scalar expansion and the volume of the model can be obtained as

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-\frac{(\sigma e^{\lambda t}+\tau e^{-\lambda t})^2}{(\sigma e^{\lambda t}-\tau e^{-\lambda t})^2}, \tag{30}$$

$$\theta=3H=3\frac{\dot{a}}{a}=3\left(\frac{\sigma\lambda e^{\lambda t}-\tau\lambda e^{-\lambda t}}{\sigma e^{\lambda t}+\tau e^{-\lambda t}}\right), \tag{31}$$

$$V=a^3=\left(\sigma e^{\lambda t}+\tau e^{-\lambda t}\right)^3. \tag{32}$$

The Null Energy Condition (NEC), Strong Energy Condition (SEC) and Dominant Energy Condition (DEC) for the linear combinations of exponential scale factor can be respectively calculated as

$$\rho+p=-\frac{2}{\kappa}\dot{H}=\frac{-8\sigma\tau\lambda^2}{\kappa(\sigma e^{\lambda t}+\tau e^{-\lambda t})^2}, \tag{33}$$

$$\rho+3p=\frac{6}{\kappa}(\dot{H}+H^2)=-\frac{6}{\kappa}\lambda^2, \tag{34}$$

$$\rho-p=\frac{2}{\kappa}[\dot{H}+3H^2]=\frac{2\lambda^2}{\kappa}\left[\frac{3\sigma^2e^{2\lambda t}+3\tau^2e^{-2\lambda t}-2\sigma\tau}{(\sigma e^{\lambda t}+\tau e^{-\lambda t})^2}\right]. \tag{35}$$

The state finder diagnostic pair $(r,\ s)$ can be calculated as

$$r=\frac{\ddot{H}}{H^3}-(2+3q)=\frac{a^2\dddot{a}}{\dot{a}^3}=\left(\frac{\sigma e^{\lambda t}+\tau e^{-\lambda t}}{\sigma e^{\lambda t}\tau e^{-\lambda t}}\right)^2, \tag{36}$$

$$s=\frac{r-1}{3(q-0.5)}=\frac{2\left[\dddot{a}a^2-\dot{a}^2\right]}{3\dot{a}\left[-2a\ddot{a}-\dot{a}^2\right]}=\frac{-8\sigma\tau}{3(\sigma e^{\lambda t}+\tau e^{-\lambda t})^2}. \tag{37}$$

It can be noted here that if the diagnostic pair $(r,\ s)$ approaches to $(1,\ 0)$ at late time of the cosmic evolution, the model is said to be Λ_{CDM} model. Now the other geometrical diagnostic $Om(z)$ diagnostic can be calculated as

$$Om(z)=\frac{E^2(z)-1}{(1+z)^3-1}=\frac{\dot{z}^2-H_0(1+z)^2}{H_0^2(1+z)^2\left[(1+z)^3-1\right]}, \tag{38}$$

$$whereE(z)=-\frac{\dot{z}}{H_0(1+z)}=\frac{\dot{a}}{H_0a}=\frac{\sigma\lambda e^{\lambda t}-\tau\lambda e^{-\lambda t}}{H_0(\sigma e^{\lambda t}+\tau e^{-\lambda t})}.$$

The throat radius for this case can be obtained a

$$R(t)=-\frac{k(\sigma e^{\lambda t}+\tau e^{-\lambda t})}{2c\lambda(\sigma e^{\lambda t}-\tau e^{-\lambda t})}. \tag{39}$$

It can be noted that when $t\to 0$, then $R\to -\frac{k(\sigma+\tau)}{2c\lambda(\sigma-\tau)}$ whereas when $t\to\infty$, then $R\to -\frac{k}{2c\lambda}$. Also R vanishes for $2\lambda t=ln\left(-\frac{\tau}{\sigma}\right)$.

3.3 Bouncing scale factor

Before the onset of inflation where the universe undergoes an almost exponential expansion, singularity occurs and therefore inflationary scenario fails to reconstruct the complete past history of the universe. As a solution to the challenges faced by the inflationary scenario, [11,12] proposed the matter bounce scenario. In the matter bounce cosmology, the initial singularity can be suitable avoided. In this scenario, matter dominates the universe at the bounce epoch and therefore it is possible to generate the density fluctuation comparable to that of observations [13, 14]. In bouncing cosmologies there is an initial phase of contraction where matter dominates the universe followed by a bounce without any singularity and then there is a causal generation for fluctuation. In this section, we will investigate the bouncing model in the presented framework to explore the geometrical degrees of freedom to explain the late time cosmic speed up phenomenon This will also enable us to investigate the bouncing behavior at an initial epoch. In order to investigate the dynamics under the purview of bouncing scenario, we have considered the dynamics of the models through the presumed bouncing scale factor, in the form $a(t) = a_0 e^{(t-t_0)^{2n}}$, where $a_0 > 0,\ n \neq 0$. Subsequently the hubble parameter can be obtained as $H = 2n(t-t_0)^{2n-1}$. As in the previous cases, in this case also, we can derive the matter pressure p, energy density ρ and EoS parameter $\omega = \frac{p}{\rho}$ from (12) – (13) as

$$p = -\frac{1}{\kappa}[2\dot{H}+3H^2] = -\frac{4n}{\kappa}(t-t_0)^{2n-2}[(2n-1)+3n(t-t_0)^{2n}], \tag{40}$$

$$\rho = \frac{3}{\kappa}(4n^2(t-t_0)^{4n-2}), \tag{41}$$

$$\omega = -\frac{(t-t_0)^{-2n}}{3n}[(2n-1)+3n(t-t_0)^{2n}]. \tag{42}$$

The NEC, SEC and DEC for the bouncing scale factor can be obtained as

$$\rho + p = -\frac{1}{\kappa}4n(2n-1)(t-t_0)^{2n-2}, \tag{43}$$

$$\rho + 3p = -\frac{6}{\kappa}[4n^2(t-t_0)^{4n-2} + 2n(2n-1)(t-t_0)^{2n-2}], \tag{44}$$

$$\rho - p = \frac{4}{\kappa}[n(2n-1)(t-t_0)^{2n-2} + 6n^2(t-t_0)^{4n-2}]. \tag{45}$$

The physical parameters such as the deceleration Parameter, scalar expansion and the volume can be calculated for the proposed bouncing scale factor as

$$q = -\left[1 + \frac{(2n-1)(t-t_0)^{-2n}}{2n}\right], \tag{46}$$

$$\theta = 6n(t-t_0)^{2n-1}, \tag{47}$$

$$V = a_0^3 e^{3(t-t_0)^{2n}}. \tag{48}$$

The parameters of geometrical diagnostics such as state finder pair $(r,\ s)$ and the $om(z)$ can be derived as

$$r = 1 + \frac{3(2n-1)(t-t_0)^{-2n}}{2n} + \frac{(2n-1)(2n-2)(t-t_0)^{-4n}}{4n^2}, \tag{49}$$

$$s = \frac{-(2n-1)(t-t_0)^{-2n}\left[6n + (2n-2)(t-t_0)^{-2n}\right]}{6n\left[(2n-1)(t-t_0)^{-2n} - 3n\right]} \tag{50}$$

and

$$Om(z) = \frac{E^2(z) - 1}{(1+z)^3 - 1} = \frac{\dot{z}^2 - H_0(1+z)^2}{H_0^2(1+z)^2\left[(1+z)^3 - 1\right]}, \tag{51}$$

$$\text{where } E(z) = -\frac{\dot{z}}{H_0(1+z)} = \frac{\dot{a}}{H_0 a} = \frac{2n(t-t_0)^{2n-1}}{H_0}.$$

The throat radius for the bouncing scale factor can be calculated as

$$R = -\frac{\kappa a}{2c\dot{a}} = -\frac{\kappa}{4nc(t-t_0)^{2n-1}}. \tag{52}$$

It can observed that, when $t \to 0$, $R = -\frac{\kappa}{4nc(-t_0)^{2n-1}}$ and when $t \to \infty$, $R \to 0$.

4. Stability analysis

In order to claim that our constructed cosmological models are stable, we will undertake the stability analysis of the cosmological models presented in the previous section. This can be obtained by analyzing $C_s^2 = \frac{dp}{d\rho}$. It is worthy to mention here that if $C_s^2 > 0$, the model is stable and for $C_s^2 < 0$, the model is unstable. Hence we can write

$$\frac{dp}{d\rho} = \frac{12\sigma\tau}{(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2((\sigma e^{\lambda t} - \tau e^{-\lambda t})^2 - 1)(1 - (\sigma e^{\lambda t} + \tau e^{-\lambda t})^2)}, \tag{53}$$

$$\frac{dp}{d\rho} = \frac{3n(t-t_0)^{2n}}{(n-1) + 3n(t-t_0)^{2n}}. \tag{54}$$

5. Luminosity distance

In this section, we have discussed the cosmological parameters by calculating the Luminosity distance. This can be calculated as

$$d_L(z) = r_1(1+z)a_0 = a_0(1+z)\int \frac{dt}{a}. \tag{55}$$

The luminosity distance for quasi linear scale factor, linear combination of exponential scale factor and bouncing scale factor can be respectively calculated as

$$d_L(z) = a_0(1+z)\int \frac{dt}{e^{H_0 t} - \frac{M^2 t^2}{12}}, \tag{56}$$

$$d_L(z) = a_0(1+z)\int \frac{dt}{\sigma e^{\lambda t} + \tau e^{-\lambda t}}, \tag{57}$$

$$d_L(z) = a_0(1+z)\int \frac{dt}{a_0 e^{(t-t_0)^{2n}}}. \tag{58}$$

The distance modulus can be calculated as

$$\mu(z) = 5\ log\ d_L(z) + 25. \tag{59}$$

The distance modulus for quasi linear scale factor, linear combination of exponential scale factor and bouncing scale factor can be respectively calculated as

$$\mu(z) = 5\ log\left[a_0(1+z)\int \frac{dt}{e^{H_0 t} - \frac{M^2t^2}{12}}\right] + 25, \tag{60}$$

$$\mu(z) = 5\ log\left[a_0(1+z)\int \frac{dt}{\sigma e^{\lambda t} + \tau e^{-\lambda t}}\right] + 25, \tag{61}$$

$$\mu(z) = 5\ log\left[a_0(1+z)\int \frac{dt}{a_0 e^{(t-t_0)^{2n}}}\right] + 25. \tag{62}$$

6. Conclusion

The physical parameters of the cosmological models are derived using the scale factors (i) quasi linear (ii) linear combination of exponential function, (iii) bouncing scale factors. The equation of state parameter, from where the nature of universe during evolution would be known, has been derived with respect to the cosmic time. The energy conditions for these three models are derived along with the physical parameters such as deceleration parameter, scalar expansion, volume, state finder pair are also derived with respect to the cosmic time. The stability analysis are performed along with the determination of luminosity distance. As a future work to this project, the graphical representation of the parameters can be presented and its behavior can be analyzed.

Acknowledgement

EG and SP are grateful to Prof. B. Mishra for his guidance to formulate as well as work out the problem.

References

1. S. Perlmutter et al., Nature **391** (1998) 51.
2. A.G. Reiss et al., Astron. J. **116** (1998) 1009.
3. R. Knop et al., Astrophys. J. **598** 102 (2003).
4. D.J. Eisenstein et al., Astrophys. J. **633** 560 (2005).
5. D.N. Spergel et al., Astrophys. J. S. **170** 377 (2007).
6. M. Sullivan et al., Astrophys. J. **737** 102 (2011).
7. N. Suzuki et al., Astrophys. J. **746** 85 (2012).
8. P.A.R. Ade et al., (Planck Collaboration) arXiv:1303.5076 [astro-ph.CO].
9. P.A.R. Ade et al., (Planck Collaboration) arXiv:1303.5082 [astro-ph.CO].
10. P.A.R. Ade et al., (Planck Collaboration) arXiv:1303.5084 [astro-ph.CO].
11. R. Brandenberger, Int. J. Mod. Phys. Conf. Ser. **01** (2011) 67.
12. R. Brandenberger, arXiv:1206.4196 (2012).
13. M. Novello and S.E. Perez Bergliaffa, Phys. Rep. **463** (2008) 127.
14. R. Brandenberger and P. Peter, Found. Phys. **47** (2017) 797.