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Bose-Einstein Condensation in a Uniformly Accelerated Frame

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In this article we have investigated the possibility of Bose-Einstein Condensation (BEC) in a frame undergoing uniform acceleration or in other wards, in Rindler space associated with the uniformly accelerated frame. We have followed a very simple conventional technique generally used in text book level studies. It has been observed that the critical temperature for BEC increases with the increase in magnitude of acceleration of the frame. Typically the critical temperature in an accelerated frame is of the order of the Unruh temperature. Hence we have concluded that the increase in the magnitude of acceleration of the frame facilitates the formation of condensed phase

Keywords: Bose condensation; Rindler space; Uniformly accelerated frame; Principle of equivalence

1. Introduction

Since bosons do not obey Pauli exclusion principle, the total wave function for a system consisting of a number of bosons is symmetric in nature. Then at low temperature, less than a typical value known as the critical temperature for BEC, a substantial number of bosons will go to the ground state energy level or zero energy level [1,2]. In a certain sense, the BEC is akin to the familiar process of liquid-vapor phase transition. Conceptually, however, the two process are very different. Unlike the liquid-vapor transition, which occured in configuration space, the BEC takes place in momentum space-described as a condensation in momentum space [3]. However with the critical examination of the equation of states of non-BEC and BEC phases shows the first order nature of the phase transition [3]. The term momentum space condensation is the thermodynamic manifestation that the transition to BEC state occurs only because of the symmetric nature of the wave function and not because of any inter particle interaction [4]. This is the very reason why it takes so many years to verify it experimentally. Although it has been predicted long ago [1,2], only in the year 1995 the phenomenon was experimentally verified [5,6]. The advancement of technology, in particular,the progress in low temperature physics, including the associated cryogenic technology helps a lot to conduct this experiment. Two experimental condensed matter physicist, Eric Cornell and Carl Wieman were awarded Nobel prize in the year 2001 to verify the formation of BEC. They have used a few hundreds of Rb^{87} atoms, which are bosons. These atomic bosons were trapped in a harmonic potential [3]. They have obtained BEC at around 170 Nano K.

Let us now introduce in brief the concept of Rindler space. It is associated with a frame undergoing uniform accelerated motion with respect to some inertial frame [7–10]. The Rindler space can very easily be shown to be flat in nature (curvature tensors are zero in this case). Therefore one can say that the Rindler space is a kind of flat Minkowski space where the uniform velocity of the moving frame (inertial in nature) is replaced by uniform acceleration (non inertial in nature). Now from the principle of equivalence a frame undergoing acceleration in absence of gravity is equivalent to a frame at rest in presence of a gravitational field of which the strength of gravitational field is exactly equal to the magnitude of the acceleration [10]. Therefore in the present situation it is equivalent to say that we are going to study BEC in a rest frame but in presence of a strong uniform background gravitational field.

Now it can very easily be shown that in the Rindler space the Hamiltonian of a particle of rest mass m_0 is given by [11–15] (see also [16–18])

$$H = \left(1 + \frac{\alpha x}{c^2}\right)\left(p^2c^2 + m_0^2c^4\right)^{1/2}, \tag{1}$$

where it is assumed that the frame is undergoing accelerated motion along x-axis, which is of course arbitrary in nature, indicates that the space spanned on $x-y$ plane is isotropic. The acceleration or the gravitational field α is constant within a length parameter x_0 (say). Then in our formalism we replace x in the above expression by x_0. For the sake of convenience one can use without the loss of generality the non-relativistic approximation of the above equation, given by

$$H = \left(1 + \frac{\alpha x}{c^2}\right)\left(\frac{p^2}{2m_0} + m_0c^2\right). \tag{2}$$

At this point we should mention that in quantum mechanical picture the above Hamiltonian H (both in relativistic and non relativistic scenario) is non-Hermitian. But is PT symmetric in nature, where P is the parity operator and T is the time reversal operator [19]. As a consequence the eigen spectrum of H is real in nature.

2. Basic Formalism

The effect of background gravitational field on the energy eigen value has come through the Rindler Hamiltonian H containing the term α. We have noticed that even in quantum mechanical studies in Rindler space the strong gravitational field affects the binding of the particles [17]. Now below the critical temperature T_c for the transition to BEC one can write

$$n = n_{p\neq 0} + n_{p=0}, \tag{3}$$

where n is the total number of bosons per unit volume and on the right hand side, the first term is the total number of normal bosons per unit volume and the second term is the corresponding quantity per unit volume in the BEC phase. In the usual case

$$n = \frac{1}{\lambda^3} g_{3/2}(z) + \frac{z}{1-z} \times \frac{1}{V}, \tag{4}$$

where $\lambda = h/(2\pi mkT)^{1/2}$, the thermal de-Broglie wavelength, z is the fugacity, $V = S_{yz}dx$ the volume of a small cylinder of cross sectional area S_{xy} with the corresponding length element dx and the function $g_{3/2}(z)$ is given by the infinite series

$$g_{3/2}(z) = \sum_{l=1}^{\infty} \frac{z^l}{l^{3/2}}. \tag{5}$$

Here the volume element V is completely arbitrary in which the field α is constant, however, we are not using this expression explicitly in our analysis. Therefore the arbitrariness is not going to affect our conclusion. The function $g_{3/2}(z)$ is a monotonically increasing function of z and is bounded within the domain $0 \leq z \leq 1$. At $z = 1$, $g_{3/2}(1) = 2.612$, beyond which it diverges to $+\infty$. Therefore at $T = T_c$, the critical condition [3]

$$n = \frac{1}{\lambda^3} g_{3/2}, \tag{6}$$

gives the critical temperature for BEC. Obviously at T_c, the chemical potential μ for the Bose particles is zero or in the relativistic scenario is m_0c^2, the rest mass energy. Therefore it is also quite clear that in the Rindler space, the value of μ and T_c will be completely different from that of the usual scenario. In Rindler space at $T = T_c^{(\alpha)}$ (say), where $T_c^{(\alpha)}$ is the modified critical temperature in the Rindler space and

$$\mu = \left(1 + \frac{\alpha x_0}{c^2}\right) m_0c^2 = u(\alpha) m_0 c^2, \tag{7}$$

is the modified form of chemical potential when the system is under going an uniform acceleration α. Obviously, the value of the chemical potential is large enough in the Rindler space if the magnitude of the acceleration is also quite high [14]. Of course here also one has to consider $z = 1$ and then the total number of particles per unit volume is given by

$$n = \frac{1}{u^{3/2}\lambda_\alpha^3} g_{3/2}, \tag{8}$$

where $u = \left(1 + \frac{\alpha x_0}{c^2}\right)$ and λ_α is the thermal de Broglie wavelength in the Rindler space. It is quite obvious that we get back the usual solution if $\alpha = 0$, i.e., in the inertial frame or $\alpha x_0/c^2 \ll 1$. Since n is fixed (there is also no change in volume), we have from Eq. (8) (see Eq. (41) of [14])

$$T_c^{(\alpha)} = u(\alpha) T_c = \left(1 + \frac{\alpha x_0}{c^2}\right) T_c. \tag{9}$$

This is the mathematical relation connecting the critical temperatures for BEC in the two different physical scenarios.Therefore in our simplified picture, the critical temperature for BEC transition will be quite high if the quantity $\alpha x_0/c^2 \gg 1$. Which may happen because of the large enough value of α, the strength of back ground gravitational field. Assuming a typical value $x_0 = 1KM = 10^5 cm$ and $T_c = 10^{-7}$K, We have

$$T_c^{(\alpha)} \approx \left(10^{-7} + 10^{-23}\alpha\right) K, \tag{10}$$

where the uniform acceleration of the frame is expressed in the unit cm/s^2. Therefore to affect T_c, the minimum value of uniform acceleration or the constant gravitational field must be $\approx 10^{16} cm/s^2$ and then only, the second part on the right hand side of Eq. (10) will be of the same order of magnitude as that of the first term. Let us now compare the second term with the well known Unruh temperature [20, 21], given by

$$T_U = \frac{\hbar\alpha}{2\pi c k_B}. \tag{11}$$

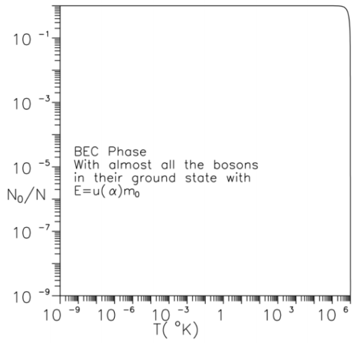

Fig. 1 Phase diagram for BEC in Rindler space (an illustration only).

The Unruh effect is associated with the quantum mechanical interaction between an accelerated observer and the inertial vacuum. Since the accelerated observer carries so much extra energy that it can transfer some of it to the inertial vacuum at the time of interaction and as a result the inertial vacuum will no longer remain vacuum. The inertial vacuum will be excited and emits radiations and particles. One can say that the vacuum state will become warmer to the accelerated observer. However, it will remain vacuum to an inertial observer at rest or in motion with uniform velocity. The expression for Unruh temperature is [20, 21]

$$T_U = \frac{\hbar\alpha}{2\pi c k_B} = 4 \times 10^{-23}\alpha/[cm/s^2][K]. \tag{12}$$

Therefore the second term on the right hand side of eqn.(10) is $\approx T_U$. Hence we may write

$$T_c^\alpha = T_c + T_U. \tag{13}$$

If $T_U \gg T_c$, we can conclude that the critical temperature $T_c^{(\alpha)}$ for BEC transition, in Rindler space is $\approx T_U$ [22]. Therefore we may conclude that in our simplified picture the uniform acceleration of the moving frame containing the Bose gas, the critical temperature for BEC can be quite high when observed from an inertial frame. It is shown in [22] that there exist a critical acceleration above which there is no BEC whereas it begins to appear in the accelerating frame when the acceleration is gradually decrease. We have noticed that in our formalism the chemical potential increases monotonically with the increase in the uniform acceleration of the moving frame. Which is consistent with the result obtained in [22]. In [23] the authors have obtained negative thermal-like correction associated with acceleration of the frame. They have used the thermo-field dynamics and the effect of uniform acceleration of the moving frame and gave an explanation for this negative sign and shown the increase in the fractional abundance of condensed phase with larger acceleration. Indirectly speaking our formalism is also consistent with the work presented in [23].

3. Conclusion

In our model the critical temperature $T_c^{(\alpha)} \approx T_U$. Therefore if the temperature of the Bose system in the uniformly accelerated frame is decreased continuously, i.e., T becomes more and more less than the corresponding critical temperature, then it is quite possible that almost all the bosons will go to the condensed phase much before absolute zero temperature. To illustrate graphically, in Fig.(1) we have plotted N_0/N against T using the expression

$$\frac{N_0}{N} = 1 - \frac{T^3}{T_c^{(\alpha)^3}}.$$

We have considered just for the sake of illustration $T_c^{(\alpha)} = 10^{7^o}$K, the core temperature of sun and vary T ($\leq T_c^{(\alpha)}$) from 10^{-7^o}K, the approximate value of the critical temperature for BEC in the laboratory to $T_c^{(\alpha)}$, the critical temperature for BEC in the Rindler space with the value of acceleration $\alpha \sim 10^{30}$cm/s^2, which is also the corresponding Unruh temperature. However, this value of the critical temperature for BEC has very little significance as the Unruh temperature. At this temperature, there is no breaking of any kind of symmetry or particle production at the core of sun. The critical temperature for BEC in the accelerated frame must therefore be several order more than 10^7K. Further, it is quite obvious from the nature of the curve that all the bosons go to their ground state $u(\alpha)m_0$ much before $T = 0$. Therefore we may conclude that the increase in the magnitude of uniform acceleration of the frame or equivalently, the strength of constant gravitational field of the rest frame facilitates the BEC process. At the end of this article we are proposing to compare our phase diagram with the standard BEC phase digram from a standard text book on Statistical Mechanics [3].

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