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Accelerating universe and anisotropic dark energy models

S.K. Tripathy[a1], A. Anand[b], Asmita Parida[c], B. Mishra[d2] and Saibal Ray[e3]

[a] *Department of Physics, Indira Gandhi Institute of Technology, Sarang, Dhenkanal 759146, Odisha, India*
[b] *Department of Computer Science Engineering and Applications, Indira Gandhi Institute of Technology, Sarang, Dhenkanal 759146, Odisha, India*
[c] *Department of Electrical Engineering, Indira Gandhi Institute of Technology, Sarang, Dhenkanal 759146, Odisha, India*
[d] *Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad 500078, India*
[e] *Department of Physics, Government College of Engineering and Ceramic Technology, Kolkata 700010, West Bengal, India*

Abstract: We discussed some accelerating anisotropic dark energy models with dynamic pressure anisotropies along different spatial directions in the framework of General Relativity. A spatially homogeneous but anisotropic LRSBI space time is considered to model the universe. Explicit expressions for directional pressure anisotropies are obtained in terms of the deceleration parameter. This provides us an opportunity to tune the evolutionary aspect of the pressure anisotropies through the evolving nature of the deceleration parameter. It is found that, for models predicting constant deceleration parameter, the pressure anisotropies are maintained throughout the cosmic evolution. However, for models simulating a signature flipping deceleration parameter, the pressure anisotropies along the symmetry axis and symmetry plane are found to evolve dynamically and continue along with the cosmic expansion.

Keywords: general relativity; dark energy; anisotropy; hybrid scale factor; cosmology

1 Introduction

The most striking and intriguing aspect of modern cosmological theories is to explain the outcome of a lot of observational data gathered over the last two decades suggesting an accelerated expansion of the universe at least at its late phase of evolution [1,2]. Also, there have been observational supporting evidences on the belief that, the expansion rate based on local data is different than the past expansion rate [3]. Lot of theoretical explanations and concepts have been developed so far to address this late time cosmic speed up issue. Also, these developments have raised many questions and doubts. Einstein's General Relativity (GR) is unable to provide suitable answers to these questions concerning the cosmic acceleration at late phase. Therefore, a negligibly small but positive cosmological constant is put by hand in the field equations of GR to get a possible accelerating scenario. However,

[1] Email: tripathy_ sunil@rediffmail.com
[2] Email: bivu@hyderabad.bits-pilani.ac.in
[3] Email: saibal@associates.iucaa.in

within the purview of GR, it is believed that the dark sector of the universe is responsible for the ongoing cosmic dynamics. The dark sector is comprising of the dark matter and the dark energy. The dark energy is believed to have played a role in the late time accelerating dynamics. The dark energy still remains as a mystery besides the known fact that, it violates the strong energy condition and resembles a cosmic fluid with negative pressure. On the other hand, the dark matter is believed to be controlled by a weakly interacting massive particle (WIMP) [4,5]. There are also views on the dark matter as a manifestation of the GR modification [6]. The dark matter problem has long been a challenging issue for the theoretical physicists. On the experimental front, concerted efforts are being made to identify dark matter particles with masses of the order of hundreds of GeV scale. But till date, no such dark matter particles found from observations and experiments [4].

As per the observations, the Universe is mostly flat and isotropic and can be well explained by the standard ΛCDM model. However, certain observational data gathered in the last decade suggest a possible departure from the global isotropy. The high resolution cosmic microwave background (CMB) radiation data from Wilkinson Microwave Anisotropy Probe (WMAP) showing some large angle anomalies [7,8,9,10], a slight redshift of the primordial power spectrum of curvature perturbation from exact scale invariance as provided by the Planck data [11,12], observation of cosmic anisotropy due to unidirectional anisotropy of cosmic ray flow along the Galactic arms [13] seem to imply a violation of statistical isotropy. Other anomalies such as the lack of correlations on large angular scales, the hemi-spherical power asymmetry and the quadrupole-octupole alignment suggest a non-trivial topology of the large scale geometry of the Universe with an asymmetric expansion. Searches for large scale anisotropies are conventionally made by looking for non-uniformities in the distribution of events in right ascension [14,15].

Recently, from a maximum-likelihood analysis of the Joint Light Curve analysis catalogue of type Ia supernovae Colin et al. [16] found that, the deceleration parameter has a bigger dipole aligned with the cosmic microwave background dipole which rejects the statistical isotropy at 3.9σ statistical significance level. These observations obviously hint towards the scale invariance of the primordial perturbations and the possible presence of some anisotropic energy source in the universe with anisotropic pressure. In recent times, there have been some anisotropic models proposed to address the issue of the smallness in the angular power spectrum and departure from the global statistical isotropy [17,18,19,20] which bear a similarity to the Bianchi morphology [21,22,23].

In the present work, we have considered an anisotropic Universe where the dynamics is governed through dark energy. The dark energy pressure is assumed to be different along different spatial directions. This assumption stems from the fact of the anisotropy present in cosmic acceleration and CMB temperature anisotropy. Also, the possibility of primordial magnetic field may provide some sort of anisotropy to the model. The dynamical evolution of the pressure anisotropies have been studied in recent times by considering different space times and models [24, 25, 26, 27, 28, 29, 30]. Therefore, our interest in the present work is to investigate the dynamical behaviour of the pressure anisotropies in an anisotropic Locally Rotationally Symmetric Bianchi I (LRSBI) metric and to correlate their dynamics to the evolutionary aspect of the deceleration parameter.

The paper is organized as follows: In Section 2, the basic formalism for anisotropic dark energy model with anisotropic pressures along different spatial directions have been discussed for an anisotropic and spatially homogeneous LRSBI metric in the framework of General Relativity. The explicit expressions of the directional pressure anisotropies have been derived in terms of the Hubble parameter and deceleration parameter. In Section 3, we have considered some accelerating models and discussed the time evolution aspect of the pressure anisotropies. At the end, the summary and conclusions of the work are presented in Section 4.

2 Basic formalism

We consider an anisotropic dark energy model with anisotropic pressures along different spatial directions in the field equations in General Relativity, $G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}$, where the energy momentum tensor for dark energy is assumed as

$$\begin{aligned} T_{ij} &= diag[\rho, -p_x, -p_y, -p_z] \\ &= diag[1, -\omega_x, -\omega_y, -\omega_z]\rho \\ &= diag[1, -(\omega+\delta), -(\omega+\gamma), -(\omega+\gamma)]\rho. \end{aligned} \tag{1}$$

The skewness parameters δ, γ are the respective deviations along $x-$ and $y-$ axes from the equation of state (EoS) parameter ω. We assume same pressure anisotropy along $y-$ and $z-$ axes. We allow these skewness parameters to evolve with the cosmic dynamics. ρ is the energy density and the pressure $p = \omega\rho$. Here we have used the gravitational units ($8\pi G = c = 1$). The line element for LRSBI space-time is considered in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2), \tag{2}$$

where the directional scale factors $A = A(t)$ and $B = B(t)$ are functions of cosmic time only. Einstein field equations for the metric (2) are

$$\frac{2\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 = \rho, \tag{3}$$

$$\frac{2\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 = -(\omega+\delta)\rho, \tag{4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -(\omega+\gamma)\rho. \tag{5}$$

An overhead dot on a field variable denotes differentiation with respect to time t. The energy conservation for the anisotropic fluid, $T^{ij}_{;j} = 0$, yields

$$\dot{\rho} + 3\rho(\omega+1)H + \rho(\delta H_x + 2\gamma H_y) = 0, \tag{6}$$

where the directional Hubble rates are defined as $H_x = \frac{\dot{A}}{A}$ and $H_y = \frac{\dot{B}}{B}$ and the mean Hubble rate is $H = \frac{1}{3}(H_x + 2H_y)$.

The above Eq. (6) can be split into two parts: the first one corresponds to the conservation of matter field with equal pressure along all the directions i.e. the deviation free part of (6) and the second one corresponds to that involving the deviations of EOS parameter:

$$\dot{\rho} + 3\rho(\omega+1)H = 0, \tag{7}$$

and

$$\rho(\delta H_x + 2\gamma H_y) = 0. \tag{8}$$

It is now certain that, the behaviour of the energy density ρ is controlled by the deviation free part of EOS parameter whereas the anisotropic pressures along different spatial directions can be obtained from the second part of the conservation equation. From Eq. (7), we obtain the energy density for a constant EOS parameter ω as $\rho = \rho_0 \mathcal{R}^{-3(\omega+1)}$, where ρ_0 is the value of energy density at the present epoch and $\mathcal{R}$ is the scale factor of the universe.

The scalar expansion θ and shear scalar σ^2 in the model are expressed as

$$\theta = (H_x + 2H_y), \tag{9}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\Sigma H_i^2 - \frac{1}{3}\theta^2\right), \tag{10}$$

where $H_i; i = 1,2,3$ are the respective directional Hubble rates along $x-$, $y-$ and $z-$ axes. Also, $\sigma_{ij} = \frac{1}{2}(u_{i;k}h_j^k + u_{j;k}h_i^k - \frac{1}{3}\theta h_{ij})$ and $h_{ij} = g_{ij} - u_i u_j$ is the projection tensor. $u_i = \delta_i^0$ is the four velocity vector in the comoving coordinates. The shear scalar is usually considered to be proportional to the scalar expansion for spatially homogeneous metrics which leads to an anisotropic relationship among the directional scale factors A and B as $B = A^k$ [31,24]. Here k is a positive constant and represents the anisotropy in the model. The anisotropic relation can also be expressed as $H_y = kH_x$. Then the mean Hubble parameter becomes

$$H = \epsilon H_x, \tag{11}$$

where $\epsilon = \frac{3}{2k+1}$. The new parameter ϵ is a positive constant. Also it takes care of the anisotropic nature of the model. If ϵ is 1 than the model is isotropic and in all other case the model retains the anisotropic behaviour. One can note that, for $k = 1$, we have $\epsilon = 1$.

From Eq. (8) we obtain

$$\delta = -\left(\frac{3-\epsilon}{\epsilon}\right)\gamma. \tag{12}$$

From Eqs. (4), (5) and (12), we get the expressions for the pressure anisotropies along different directions as

$$\delta = \frac{(1-\epsilon)(3-\epsilon)}{2}\frac{F(H)}{\rho}, \tag{13}$$

$$\gamma = \frac{\epsilon(\epsilon-1)}{2}\frac{F(H)}{\rho}. \tag{14}$$

where $F(H) = \left(\dot{H} + 3H^2\right)$.

In GR, this functional $F(H)$ has a great role in ascertaining accelerating models [32]. If the functional $F(H)$ vanishes then accelerating models can not be achieved for LRSBI metric. This fact has already been shown in some earlier works [32,25,33]. However, in presence of some anisotropic sources such as the anisotropic dark energy components along different directions, we may obtain a non vanishing functional $F(H)$ and therefore accelerating models can be well constructed [25,33].

The dynamical behaviour of the pressure anisotropies δ and γ is decided by the behaviour of the factor $\frac{F(H)}{\rho}$. This requires the behaviour of the energy density. The limiting value of the anisotropic parameter ϵ can be obtained from an analysis of the energy density. The expression for the energy density can be obtained from Eq. (3) as

$$\rho = \frac{3}{4}(3-\epsilon)(\epsilon+1)H^2. \tag{15}$$

The energy density of the universe should be positive throughout the cosmic evolution. In this sense, we have from Eq. (15), $\rho > 0$ for $\epsilon < 3$. Since $\epsilon = \frac{3}{2k+1}$, the limiting value k assumes is $k > 1$.

The factor $\frac{F(H)}{\rho}$ is evaluated as

$$\frac{F(H)}{\rho} = \frac{4}{3(3-\epsilon)(\epsilon+1)}(2-q), \tag{16}$$

where $q = -1 - \frac{\dot{H}}{H^2}$ is the deceleration parameter.

In terms of the deceleration parameter, the pressure anisotropies may be expressed as

$$\delta = \frac{2}{3}\left(\frac{1-\epsilon}{1+\epsilon}\right)(2-q), \tag{17}$$

$$\gamma = \frac{2}{3}\frac{\epsilon(\epsilon-1)}{(3-\epsilon)(\epsilon+1)}(2-q). \tag{18}$$

The above equations clearly indicate that, in an LRSBI dark energy dominated universe, the pressure anisotropies along different spatial directions depend on the evolutionary aspect of the deceleration parameter. If q is evolving with time, then the pressure anisotropies will evolve otherwise we get constant pressure anisotropies. In order to get an idea of the total anisotropies, we may take a sum

$$\delta + \gamma = \frac{2}{3}\frac{(1-\epsilon)(3-2\epsilon)}{(3-\epsilon)(\epsilon+1)}(2-q). \tag{19}$$

3 Some accelerating models

In this section, we wish to investigate some accelerating models by considering some well known forms of the Hubble parameter. Our interest is to study the evolutionary behaviour of the anisotropic dark energy pressure along different spatial directions. The commonly known accelerating models as available in literature are the de Sitter model (H=constant), the power law expansion of the scale factor and the hybrid scale factor.

3.1 de Sitter model (H=constant)

As a first case, we consider the de Sitter model where the universe expands exponentially with time. The de Sitter expansion model is represented through a constant Hubble parameter

$$H = H_0, \tag{20}$$

where H_0 is a constant and is equal to the Hubble parameter as measured in the present epoch. Obviously for this model, the deceleration parameter becomes $q = -1 - \frac{\dot{H}}{H^2} = -1$.

We can obtain the expression for the functional $F(H)$ for the de Sitter model as

$$F(H) = 3H_0^2. \tag{21}$$

The energy density ρ may be obtained as

$$\rho = \frac{3}{4}(3-\epsilon)(\epsilon+1){H_0}^2. \tag{22}$$

Since the Hubble parameter is a constant quantity, the dark energy density for this model becomes a constant quantity. It should be recalled here that we have considered only dark energy as the contributing factor to the matter field. Along with the dark energy, we have considered some unknown sources for anisotropic dark energy pressure along different spatial directions.

The pressure anisotropies along different spatial directions such as δ and γ are obtained as

$$\delta = \frac{3}{2\rho}(1-\epsilon)(3-\epsilon)H_0^2, \tag{23}$$

$$\gamma = \frac{3}{2\rho}\epsilon(\epsilon-1)H_0^2. \tag{24}$$

Since the energy density can be expressed in terms the Hubble parameter, we may write the pressure anisotropies as

$$\delta = 2\left(\frac{1-\epsilon}{1+\epsilon}\right), \tag{25}$$

$$\gamma = -2\left(\frac{\epsilon}{3-\epsilon}\right)\left(\frac{1-\epsilon}{1+\epsilon}\right). \tag{26}$$

It is interesting to note that, the pressure anisotropies come out to be constant quantities for this model. In other sense, if we enforce a constant anisotropic dark energy source which results in a non evolving dark energy density leads to constant finite pressure anisotropies along different spatial directions. Also, for such a situation, the pressure anisotropy is maintained throughout the cosmic evolution for the de Sitter model. In order to get a quantitative view of the pressure anisotropies, we may consider some representative values for the anisotropic parameter ϵ namely $\epsilon = 0.9$ and 0.95. For these representative values of ϵ we get δ respectively as 0.105 and 0.051. Similarly, the pressure anisotropy along yz plane is obtained to be -0.045 and -0.024 respectively. With an increase in the value of ϵ, the pressure anisotropy along the x-axis decreases but the pressure anisotropy along the yz plane increases. Another fact is that, while δ becomes a positive quantity, we obtain γ as a negative quantity.

The sum of the pressure anisotropies for the present model become

$$\delta + \gamma = 2\frac{(1-\epsilon)(3-2\epsilon)}{(3-\epsilon)(1+\epsilon)}. \tag{27}$$

For the representative values of the anisotropic parameter ϵ, we have sum of the pressure anisotropies as 0.06 and 0.027. With an increasing value of ϵ towards

1, it may be found that, the sum of the pressure anisotropies vanishes.

3.2 Power law expansion of the scale factor

Here we consider a power law expansion model of the universe where the scale factor of the model increases as $\mathcal{R} = t^m$, m being a positive constant. Power law expansion models are very popular in addressing many issues in cosmology. The Hubble parameter for this model can be expressed as

$$H = \frac{m}{t}. \tag{28}$$

The deceleration parameter for power law expansion model becomes

$$q = -1 + \frac{1}{m}. \tag{29}$$

An accelerating model is characterized by a negative deceleration parameter. This fact restricts the parameter m in the range $m > 1$.

For the power law expansion model, the energy density may be calculated as

$$\rho = \frac{3m^2}{4} \frac{(3-\epsilon)(\epsilon+1)}{t^2}, \tag{30}$$

which is obviously positive for a choice of $\epsilon < 3$ and $m > 1$.

The functional $F(H)$ for the present power law expansion model becomes

$$F(H) = \frac{m}{t^2}(3m-1), \tag{31}$$

and consequently, the directional pressure anisotropies γ and δ would be

$$\delta = \frac{2}{3}\left(\frac{1-\epsilon}{1+\epsilon}\right)\left(3 - \frac{1}{m}\right), \tag{32}$$

$$\gamma = \frac{2}{3}\frac{\epsilon(\epsilon-1)}{(3-\epsilon)(\epsilon+1)}\left(3 - \frac{1}{m}\right). \tag{33}$$

In this case also, we get constant pressure anisotropies along different spatial directions. The sum of the pressure anisotropies becomes

$$\delta + \gamma = \frac{2}{3}\frac{(1-\epsilon)(3-2\epsilon)}{(3-\epsilon)(\epsilon+1)}\left(3 - \frac{1}{m}\right). \tag{34}$$

In order to get a quantitative view of the pressure anisotropies, we choose some representative values of the anisotropic parameter ϵ and the model parameter m. As before, we consider $\epsilon = 0.9$ and 0.95. For m, we have a restriction on it such as $m > 1$. This is required to get an accelerating model. In view of this, we chose $m = 1.1$. For $\epsilon = 0.9$, we get $\delta = 0.073$ and $\gamma = -0.031$. For $\epsilon = 0.95$, we obtain $\delta = 0.036$ and $\gamma = -0.017$. With an increase in ϵ, while there is an increase in γ, there is a decrease in the value of δ.

3.3 Hybrid scale factor

The late time cosmic speed up issue has triggered many novel ideas and concept. Within the purview of the GR, the exotic dark energy is an obvious choice to explain the late time cosmic acceleration. Another important issue concerning the late time cosmic acceleration phenomena is that whether the Universe has undergone a transition from an early deceleration to late time acceleration. If so, then what is the time frame within which such a transition has occurred? Usually, the acceleration or deceleration of the cosmic expansion is assessed through the deceleration parameter q. If $q < 0$, the model is assumed to be accelerating on the other hand if $q > 0$ the model is considered to be decelerating. Basing upon the analysis of the deceleration parameter, we may define a transition redshift z_t at which the deceleration parameter becomes zero i.e. $q = 0$. It is believed that, if at all there has occurred a late time cosmic acceleration, then the transition redshift z_t may be regarded as a fundamental constant parameter! In many studies, this quantity has been constrained in order of unity, i.e. $z_t \sim 1$. One interesting similarity of the power law expansion model or the de Sitter expansion model is that, both may provide accelerating models but with a non-evolving deceleration parameter. In the context of a transitioning universe, it is required that the deceleration parameter should be evolving in nature with a signature flipping behaviour. Such a scenario may be simulated through the concept of a hybrid scale factor (HSF) defined through a Hubble parameter [33]

$$H = \alpha + \frac{\beta}{t}, \tag{35}$$

where α and β are positive constants. The deceleration parameter for the HSF model may be expressed as $q = -1 + \frac{\beta}{(\alpha t+\beta)^2}$. Considering suitable parameters of the HSF model, it is possible to model transitioning universe with early deceleration and late time acceleration.

For the HSF model, we obtain the energy density as

$$\rho = \frac{3}{4}(3-\epsilon)(\epsilon+1)\left(\alpha + \frac{\beta}{t}\right)^2, \tag{36}$$

and the functional $F(H)$ as

$$F(H) = \frac{\beta(3\beta-1)}{t^2} + \frac{6\alpha\beta}{t} + 3\alpha^2. \tag{37}$$

The expressions of the directional pressure anisotropies γ and δ become

$$\delta = \frac{2}{3}\left(\frac{1-\epsilon}{1+\epsilon}\right)\left(3 - \frac{\beta}{(\alpha t+\beta)^2}\right), \tag{38}$$

$$\gamma = \frac{2}{3}\frac{\epsilon(\epsilon-1)}{(3-\epsilon)(\epsilon+1)}\left(3 - \frac{\beta}{(\alpha t+\beta)^2}\right). \tag{39}$$

It is interesting to note here that, the pressure anisotropies in this HSF model are evolving with time. This fact has come from the time evolution of the deceleration parameter. In order to understand the time evolution of the pressure anisotropies in HSF model, we have considered some recently constructed HSF models with parametric value as given in the Table 1 [34,35]. These models have been constructed from the observational $H(z)$ data.

In Fig. 1(a) and (b), the time evolution of the parameters representing the pressure anisotropies along different spatial directions are shown for the four models

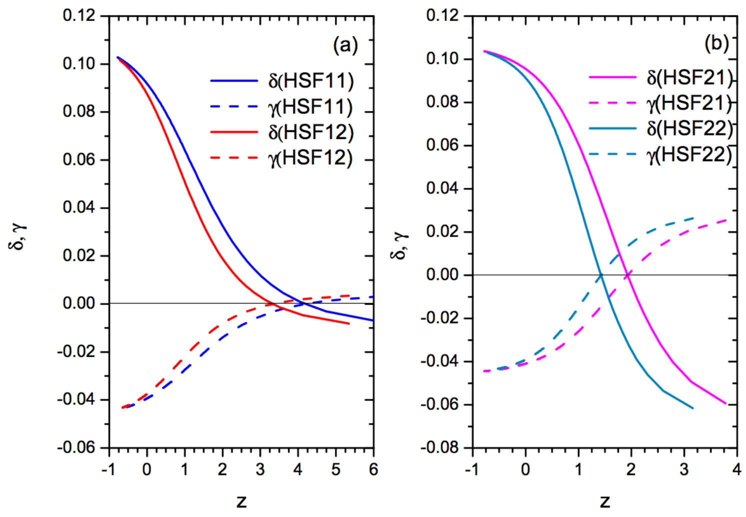

Fig. 1 (a) The pressure anisotropies δ and γ for the hybrid scale factor models HSF11 and HSF12, (b) same as (a) for the models HSF21 and HSF22. Here we have considered the anisotropic parameter as $\epsilon = 0.9$.

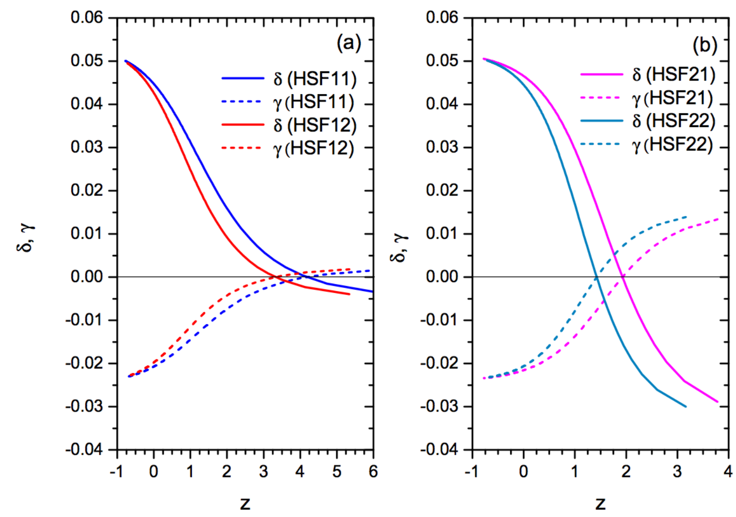

Fig. 2 (a) The pressure anisotropies δ and γ for the hybrid scale factor models HSF11 and HSF12, (b) same as (a) for the models HSF21 and HSF22. Here we have considered the anisotropic parameter as $\epsilon = 0.95$.

Table 1 Model parameters of the hybrid scale factor [34, 35].

Models	β	α	z_t	z_r
HSF11	0.3	0.585	0.8	4.05
HSF21	0.2	0.65	0.8	1.925
HSF12	0.3	0.47	0.5	3.517
HSF22	0.2	0.51	0.5	1.448

where we have considered $\epsilon = 0.9$. The same has been repeated in Fig. 2(a) and (b) for $\epsilon = 0.95$. The upper solid curves represent the evolution of δ and the lower dashed curves represent the evolutionary aspects of γ. We have considered the figures upto a reasonable value of redshift around $z = 6$. Within these time frame, we observe that, while δ increases with cosmic expansion, γ decreases. There occurs a signature reversal in the behaviour of the pressure anisotropies δ and γ. We denote the redshift at which the signature flipping in δ and γ occurs as z_r. It is found that, z_r depends on the HSF model chosen but is independent of the choice of the anisotropic parameter ϵ. The reversal redshift as obtained for the four HSF models are $z_r = 4.05, 1.925, 3.517$ and 1.448 corresponding to the HSF models HSF11, HSF21, HSF12 and HSF22. One should note that, the behavioural reversal redshift z_r is quite different than the transition redshift z_t, even though both are of the order of unity.

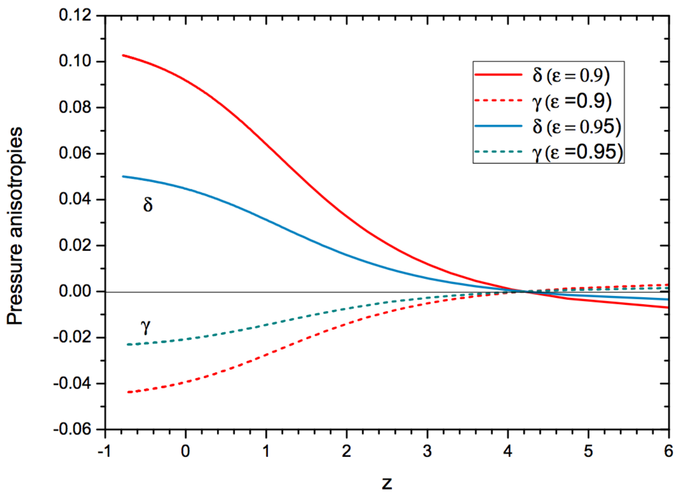

Fig. 3 The effect of ϵ on δ and γ is shown for the model HSF11.

In Fig. 3, we show the effect of the anisotropic parameter ϵ on any given HSF model within the formalism discussed in the present work. Here we have considered only the HSF11 model. It is obvious from the figure that, with an increase in the value of ϵ, the magnitudes of the pressure anisotropies decrease substantially. Also, a change in the value of ϵ does not affect the behavioural reversal redshift z_r. Similar conclusion may be inferred for all other HSF models.

4 Summary and conclusion

In the present work, we have discussed some accelerating models supported by some anisotropic dark energy. We have assumed different pressure along different spatial directions in the back drop of an anisotropic LRSBI metric. The departure of the directional pressures from the isotropic pressure are assumed to be characterized by some directional pressure anisotropies. Assuming some specific forms of accelerating Hubble parameters, we have derived the expressions of the directional pressure anisotropies and expressed them in terms of the deceleration parameter. It is shown that, the evolutionary aspects of the pressure anisotropies directly depend on the evolutionary behaviour of the deceleration parameter. Three different forms of the Hubble parameter are considered namely, the de Sitter scenario, the power law expansion and the hybrid function. Out of these three forms, the first two provide a constant deceleration parameter whereas the third function provides a signature flipping behaviour of the deceleration parameter with early epoch positive values and late epoch negative values. Since the evolutionary aspects of the pressure anisotropies are associated with the time evolution of the deceleration parameter, in the first two cases, we obtain constant pressure anisotropies which are maintained through out the cosmic evolution.

HSF is required to simulate a signature flipping deceleration parameter that provides a realistic picture of the transitioning Universe from early deceleration to late time acceleration. We have considered four different models of the HSF to investigate the evolutionary aspects of the pressure anisotropies along different directions. It is certain that, since the deceleration parameter for HSF models evolve with time, the pressure anisotropies also evolve with time. While the pressure anisotropy along the x-axis increases with cosmic evolution, the pressure anisotropy along the symmetry plane decreases with time. An interesting behaviour is seen in the evolutionary aspects of the pressure anisotropies. Both of them reverse their signatures at certain behavioural reversal redshift which depends on the model chosen. We have also examined the effect of the anisotropic parameter on the evolution of the pressure anisotropies and found that, with an increase in the value of the anisotropic parameter, the magnitudes of the pressure anisotropies decrease substantially.

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