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A note on the modified Newtonian gravity and its application

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Abstract: In the present work we propose to modify the Newtonian gravitational law by incorporating a velocity dependence term which can be augmented with the Hubble's law. We have applied the modified formula to local galactic systems to account for the recessional velocity of the galaxies and hence expanding phase of the universe. The model seems physically viable on the theoretical ground, however its future feasibility in connection to several other astrophysical observations is to be sought for.

Keywords: Newtonian gravitational law; Hubble's law; galactic systems; recessional velocity

1. Introduction

To understand the mechanism behind the motions of the planets, stars, nebulae and hence as a whole the so-called universe in the ancient time as was usually viewed by people was a challenge to the philosophers for several millenniums starting from Maya to Greek civilizations. The genuine research to unfold the mysteries of the heavenly bodies started in the medieval period when Copernicus (1473 - 1543), Brahe (1546 - 1601) and Kepler (1571 - 1630) considered the issue with scientific temperament via theory as well as observation [1]. Eventually Newton (1642 - 1726) conceived the background idea and generalized the Keplerian laws of planetary motion into a unique Law of Gravitation which is the first great unification in science [2].

This law has been received tremendous success throughout the last few centuries which reads as

$$F_N = -\frac{Gm_1m_2}{r^2}, \tag{1}$$

where F_N is the attractive force acting between two bodies of masses m_1 and m_2 at a distance r and G is the universal gravitational constant.

However, time and again discrepancies also have been pointed out by several scientists. It is stated by Ghosh and Dey [3] that in the 19th century, when mathematical tools became more matured, some serious problems were faced with this law where a number of manifestations of the paradox leading to inconsistent results, infinite potential, etc. Historically, the first modification in the law was suggested by Laplace [4] where the gravitational constant G was assumed to be exponentially varying with distance as follows:

$$F_{mod} = -F_N exp(-\Gamma r), \tag{2}$$

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where Γ denotes the intensity of attenuation of gravity.

Long back, one century ago, See [5] assumed the changes in the Newtonian law of gravitation as indicated by the various latest researches on the motions of planets and the moon to account for the Mercury's perihelion motion and other astronomical phenomena. He basically employed the formula provided by Weber [6] which is as follows:

$$F_{mod} = F_N \left[1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{c^2}\left(\frac{d^2r}{dt^2}\right)\right]. \tag{3}$$

As can be noticed, the above Eq. (3) has one small second term depending on $(dr/dt)^2$ which represents square of the velocity in the direction of the radius vector and another small third term depending on d^2r/dt^2, which is the acceleration, i.e. change of the velocity in the direction of the radius vector. This modified formula accounts for the Mercury's perihelion motion and other astronomical phenomena. However, in the present investigation we confine ourselves within the periphery of purely classical gravitation of Newton and are not considering the gravitation of Einstein, known as General Relativity, which accurately explains the Mercury's perihelion motion and acts fantastically for massive celestial bodies, e.g. compact stars, galaxies and so on.

Another modification of Newton's laws to account for observed properties of galaxies was put forward by Milgrom [7], known as the Modified Newtonian Dynamics (MOND), where the special form of the formula has been proposed as

$$F_{mod} = m_1\mu\left(\frac{a^2}{a_0}\right), \tag{4}$$

where m_1 is the object's gravitational mass, a is its acceleration, $\mu(x)$ is the interpolating function and a_0 is a new fundamental constant which marks the transition between the Newtonian and deep-MOND regimes.

In recent years many scientists have suggested different modifications of the gravitational law. It has been shown by Whitehouse and Kraniotis [8] that the flat rotation curve of galaxies may be explained by the cosmological term Λ and presence of dark matter is not necessary if Newtons gravitational equation is modified in the form

$$F_{mod} = -F_N + G_\Lambda m_2 r, \tag{5}$$

where G_Λ is the gravitational force exerted by the cosmological term Λ and represents a fifth fundamental force which is directly proportional to the distance.

On the other hand, Kirillov and Turaev [9] by using a Modified Field Theory (MOFT) have shown that the renormalization of the gravitational constant leads to the deviation of the law of gravity from the Newtons law in which the gravitational potential shows essentially logarithmic, i.e. $ln\ r$ (instead of $1/r$) behavior and hence the renormalized value of the gravitational constant G varies as increasing manner (in this connection for a detailed review on Dirac's Large Number Hypothesis (LNH) vide [10]). Based on the suggestion that the gravity is originally an entropic force a modified Newton's law of gravitation has been acheived by Ali and Towfik [11]. They argue that the modification agrees with different sign with the prediction of Randall-Sundrum II model which contains one uncompactified extra dimension and such modification may have observable consequences at length scales much larger than the Planck scale.

2. A simple modification of Newton's law of gravity

Currently our universe is expanding [12] with acceleration [13,14]. As a result, all the celestial bodies, especially the galaxies are moving away from each other. Equivalently, all the galaxies are moving away from our Galaxy, i.e. Milky Way and hence from the Earth. The Newtonian Law of Gravitation [2] works fine in our Solar system and even in short/middle distance but fails at a cosmological distance which is of the order of Megaparsec (Mpc).

Thus there is a need to modify the gravitational law to resolve this problem. However, our proposal is not completely *ad hoc* rather we are keeping in mind the expanding universe [12] and therefore propose for a modified formula which represents more general form of Newton's law of gravity and can be expressed as

$$F_{mod} = -F_N + F_N \dot{r}, \tag{6}$$

where $v = \dot{r}$ is the universal velocity.

In the above Eq. (6) we have considered that the Newtonian force acts as inverse square form of distance as well as directly proportional to velocity and can be rewritten as

$$F_{mod} = -F_N(1 - \dot{r}). \tag{7}$$

It is interesting to note that the second term in our Eq. (7) resembles with the square of the velocity term acting in the direction of the radius vector as proposed by Weber [6,5,15]. One can also note that Ghosh [16] has proposed model of Inertial Induction (as coined by Sciama [17] for the acceleration-dependent extra term) based upon an extension of Mach's Principle according to which the gravitational interaction between two masses depends not only on their distance but also on their relative velocity. It is curious that in this extended gravitational model the force law comes out as $F = ma+$ a small drag term. By using this model Ghosh [16] could quantitatively explain the cosmological redshift in a quasi-static infinite universe. Interestingly, the theory of Inertial Induction also provides feasible results regarding the dynamic gravitation phenomenon of the Earth on its satellites as a possible partial cause for orbital decay [18]. In connection to Inertial Induction it may be pointed out that not only the redshift and satellite orbit decay rather quite a few phenomena got explained by Ghosh and collaborators [19,20,21,22,23].

Now assuming that the above velocity involves in the Hubble law, i.e. $H = \dot{r}/r$, we can modify Eq. (7) as follows

$$F_{mod} = -\frac{Gm_1m_2}{r^2}(1 - Hr), \tag{8}$$

where H is the Hubble constant.

Now acceleration can be, from Eq. (8), defined by the Gravitational force on unit mass of Galaxy due to Earth as follows

$$a = \frac{F_{mod}}{m_1} = -\frac{Gm_2}{r^2}(1 - Hr). \tag{9}$$

The Newtonian Law [Eq. (1)] works fine when distance is less than 0.1 Mpc (it is to note that the average distance between galaxies is 1 to 10 Mpc). However, beyond the distance $r > 0.1$ Mpc, the gravitational pull becomes ineffective and as a result the galaxies keep moving away from each other.

3. Application of the modified law: A few test cases

3.1 The Earth-Moon system

Let us consider the following data set: $G = 6.674 \times 10^{-11}\ m^3/kg/s^{-2}$ and $1\ Mpc = 3.08 \times 10^{22}$ m, the mass of the Earth $m_1 = 6 \times 10^{24}$ kg, the mass of the Moon $m_2 = 7.342 \times 10^{22}$ kg, average distance between the Earth-Moon system $r = 3.84 \times 10^8$ m [24].

Putting these values in Eq. (8), in order to verify it, we can get the gravitational force working in between two celestial bodies. So, for the Earth and Moon system, according to the new formula we can have

$$F_{mod} = -19.83 \times 10^{19}(1 - 8.96 \times 10^{-13}) \text{ dynes.} \tag{10}$$

The second term inside the bracket is much less than the first term and so the gravitational force is negative showing the nature of it as the force of attraction. Hence as far as the above data is concerned the modified formula works fine in this case.

3.2 The Earth-Sun system

Putting the values of mass of the Sun $m_2 = 2 \times 10^{30}$ kg, distance between the Earth and Sun $R = 1.5 \times 10^{11}$ m in Eq. (8), we get the gravitational force between the Sun and Earth as

$$F_{mod} = -360 \times 10^{20}(1 - 3.5 \times 10^{-10}) \text{ dynes.} \tag{11}$$

The negative sign in Eq. (11) indicates that the resultant force is attractive between the Sun and Earth.

3.3 The Saturn-Sun system

Taking mass of the Saturn $m_1 = 5.683 \times 10^{26}$ kg, mass of the Sun $m_2 = 2 \times 10^{30}$ kg, distance between the Earth and Sun $R = 1.433 \times 10^{12}$ m, we get from Eq. (8), the gravitational force between Sun and Saturn as

$$F_{mod} = -37.9 \times 10^{21}(1 - 3.34 \times 10^{-9}) \text{ dynes.} \tag{12}$$

Here also the resultant force is attractive in nature between the Sun and Saturn.

3.4 The Earth-Virgo system

We consider her the data set as follows: mass of the Virgo $m_1 = 2.4 \times 10^{45}$ kg, distance of the Virgo from the Earth $R = 17.14$ Mpc$= 5.14 \times 10^{23}$ m. These values from Eq. (8) provide the gravitational force between Earth and Virgo as

$$F_{mod} = -3.6 \times 10^{12}(1 - 1200) \text{ dynes.} \tag{13}$$

Here the first term within bracket is very small compared to the second term. So the gravitational force between the Earth and Virgo is negative and hence repulsive in nature. Therefore the galaxy is moving away from the Earth.

The acceleration of this galaxy can be determined by using Eq. (9) as follows:

$$a = 1.5 \times 10^{-33} \times (1 - 1200)\ km/s^2. \tag{14}$$

On the other hand, according to the original Newtonian Law the acceleration of the galaxy is

$$a = 1.5 \times 10^{-33}\ km/s^2. \tag{15}$$

So, according to the new formula we can see acceleration of this galaxy is faster than that derived from the old formula by a factor (1-1200) ≈ -1200. Interestingly, the numerical value of this factor is nearly the same as the recessional velocity (v) of this galaxy where $v = Hr$, where H is the Hubbles constant as mentioned earlier.

3.5 The Earth-Corona Borealis system

The mass of Corona Borealis $m_1 = 24 \times 10^{46}$ kg, mass of the Earth $m_2 = 6 \times 10^{24}$ kg, distance of the Corona Borealis from Earth $r = 314.28 Mpc = 314.28 \times 3 \times 10^{22}$ m. Putting these values in Eq. (8), we get the gravitational force between the Earth and Virgo as

$$F_{mod} = -1.09 \times 10^{12}(1 - 22000)\ \text{dynes}. \tag{16}$$

Here the first term within the bracket is very small compared to the second term. So the gravitational force between the Earth and Corona Borealis is positive and being repulsive the galaxy is moving away from the Earth. Likewise the previous case, it is to verify that according to this formula the acceleration of the galaxy becomes faster than that derived from the original Newtonian formula by a factor (1 - 22000) ≈ - 22000 which numerically is exactly the same as the recessional velocity (v) of the galaxy under consideration.

In a similar way, one can go on to find out the gravitational force between the Earth and any other galaxy according to the modified formula. In Table 1 a list of some important galaxies with the redshift and their distance from the Earth are provided by taking the value of the Hubble constant as 70 $km/s/Mpc$ [25,26, 27] which is the accepted value at present day. However, for the observed redshift we have used data from the book by Schneider [24] where the value of the Hubble constant was available from the old source as $H = 80$ $km/s/Mpc$.

Table 1 Estimated redshift from the proposed model for $H = 70$ km/s/Mpc [25,26,27]

Galaxy	Observed distance (in Mpc)	Observed redshift (in km/sec)	Estimated redshift (in km/sec)
Virgo	17.14	1,200	1,199
Ursa Major	220	15,400	15,399
Corona Borealis	314.28	22,000	21,999
Bootes	562.85	39,400	39,399
Hydra	865.71	60,600	60,599

4. Conclusion

In this study we have presented a simple phenomenological model and modified the Newtonian gravitational law to account for the recessional velocity of galaxies.

To verify the proposed formula we have applied it arbitrarily to five galaxies and the data are included in Table 1. The results as shown in this table indicate that when the distance between two celestial objects is more than 1 Mpc the estimated redshift from our proposed modified formula fits remarkably with the observed values and this therefore admits the feasibility of our phenomenological model. To explain the expanding universe it can be argued that, as a consequence of this new law acting at a Mpc distance scale, the gravitational pull between them gradually becomes very weak and they keep increasing the recessional velocities.

Although there are striking resemblances between the observed and estimated values of the redshift for a set of galaxies, however issues are also involved in the theoretical as well as the applied fields which may be listed as follows: (i) how far the modified formula could account for the accelerating phase of the universe [13, 14], (ii) in the modified formula only the velocity term along with the distance has been considered to adapt with the expanding universe, however, following the works of Weber [6], See [5], Milgrom [7] and Ghosh [16] one can also logically approach to add the acceleration term in the formula to look for further features and (iii) specifically how the Milgrom law [Eq. (4)] due to MOND theory [7] and the allied astrophysical as well as cosmological issues can be addressed from the viewpoint of the modified Newtonian law presented in this paper. All these issues can be taken into account in future projects.

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