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Dynamic behavior of PWM Controlled DC drive

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Abstract: This paper describes bifurcation phenomena of a PWM controlled dc shunt drive. Bifurcation behavior of the system is observed by varying system parameters. The route of transition from periodic behavior to chaos has been investigated and quantified by maximum Lyapunov exponent. The stability of the system is found out using the state transition matrix over one switching cycle (the monodromy matrix) including the state transition matrices during each switching (the saltation matrices). The parameter values at which the nominal period-1 orbit loses stability is determined.

Keywords: Chaos; Bifurcations; DC drive; monodromy matrix; saltation matrix; Lyapunov exponent

1. Introduction

Chaos is an aperiodic long-term behavior of a deterministic system that exhibit sensitive dependence of initial condition. Most of the systems available around us are nonlinear in nature. But much time and energy is spent in understanding a linear system. Moreover, analysis of nonlinear system is also carried out with the help of theory developed for linear systems. This analyses the behavior of a nonlinear system by local linearization. But there are limitations of the analysis of nonlinear system in the light of linear control theory. This approximation fails to reveal actual behavior of the nonlinear system. A fascinating phenomenon of a nonlinear system is the occurrence of chaos. This behavior of a nonlinear system is not due to noise and complexity of the system. It is inherent behavior of a nonlinear system. Chaos theory provides tools to carry out this analysis.

A DC drive is extensively used in the industry. The knowledge of the behavior of the DC drive is of great importance for its proper functioning and design. Since the DC drive is nonlinear in nature, chaos occurs in this system. The occurrence of chaos and routes to chaos has been investigated in the light of chaos theory.

Chau et al. [1] investigated the nonlinear dynamics and chaotic behavior of chopper fed permanent magnet motor drive. Chakrabarty et al. [2-6] reported chaos in dc series drive by numerical simulation. A technique based on Filippov's method of differential inclusion [7] has been employed to analyze the stability of mechanical switching system [8], dc-dc converter [9], full bridge converter fed drive [10] and Photovoltaic system [11].

In this paper, the dynamic behavior of the DC shunt drive has been investigated to find out the dependence of different parameters in the system. Chaotic dynamics of the drive are then discussed. The stability analysis of the period-1 orbit of the drive is performed by the derivation of saltation and monodromy matrices of the system.

2. Mathematical model

The block diagram and equivalent circuit of the DC drive circuit is shown in the Figs. 1and 2 respectively. The output of the speed sensor (W) is compared with the reference speed (W_{ref}) in the comparator A_1. The difference of (W-W_{ref}) is compared with a ramp voltage. The output of the comparator A_2 is used to switch on the switch for the chopper drive. When control voltage exceeds ramp voltage, power switch of the chopper is off and diode is on;

otherwise, the power switch is on and diode is off. The system is linear in nature except the switch. The chaotic behavior of the system is due to switching nonlinearity.

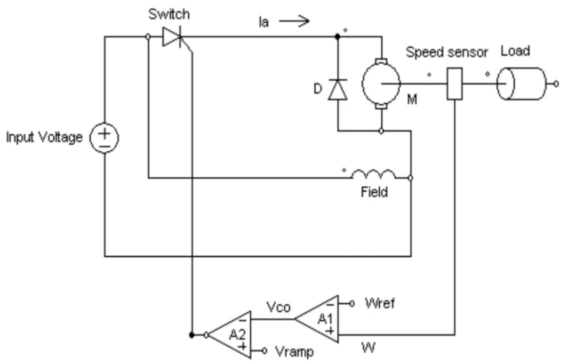

Fig.1: Schematic diagram of DC shunt drive

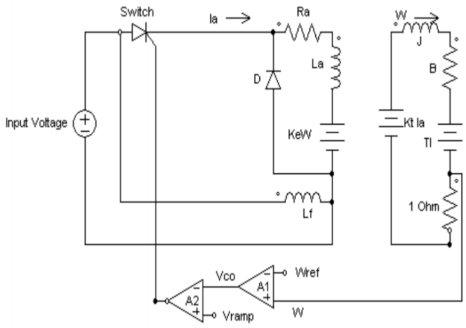

Fig. 2: Equivalent circuit of DC shunt drive

With reference to Fig. 2, i (t) is the armature current, R_a , armature resistance, L_a , armature inductance, V, supply voltage, K_E , back emf constant, K_T , torque constant, B ,viscous damping, J, load inertia and T_l load torque.

The system equations are given by

$$La\frac{di}{dt} = -R_a i - K_E\omega + V$$
$$J\frac{d\omega}{dt} = K_T i - B\omega - T_l \qquad (1)$$

Motor speed, ω , is controlled by naturally sampled constant frequency pulse width modulation. The operational amplifier *A1* has gain *G*. The control signal, $V_{co}(t) = G[\omega(t) - \omega_{ref}(t)]$ where, $\omega(t)$ and ω_{ref} are instantaneous motor speed and reference speed respectively, compared with a ramp signal V_{ramp} in the comparator A_2. The ramp is given by $V_{ramp} = V_l + (V_u - V_l)\frac{t}{T}$, where V_u and V_l are upper and lower levels of ramp having period *T*.

The chopper fed shunt motor drive operates in the continuous current mode. The system can be divided into two stages depending on the switching conditions.

The switch will be on when $V_{co}<V_{ramp}$. The corresponding system equations are

$$\frac{d}{dt}\begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} \frac{-Ra}{La} & \frac{-K_E}{La} \\ \frac{K_T}{J} & \frac{-B}{J} \end{bmatrix} \begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} \frac{V}{La} \\ \frac{-T_l}{J} \end{bmatrix} \tag{2}$$

The switch will be off when $V_{co}>V_{ramp.}$ The corresponding equations are

$$\frac{d}{dt}\begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} \frac{-Ra}{La} & \frac{-K_E}{La} \\ \frac{K_T}{J} & \frac{-B}{J} \end{bmatrix} \begin{bmatrix} i(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-T_l}{J} \end{bmatrix} \tag{3}$$

By defining the state vector x(t) and the following matrices A, E_1, E_2, The system can be rewritten as $\dot{x}(t) = Ax(t) + E_K$, where K= 1, 2. K changes the value depending on 'On' or 'Off' condition of the switch. Thus, the closed loop drive system is a second order non-autonomous dynamical system.

3. Bifurcation behavior

In order to study dynamic behavior of the DC drives, parameters of the drive system are as follows: *V_l=0, Vu=2.2, T=8ms, G=0.8, Ra=7.8Ω, La=30. mH, B=0.000654,J=0.000971,T_l=0.5 Nm, w_{ref} =100 rad/sec. K_T=0.1324, K_E=0.1356.*

The drive has been simulated with SIMULINK of MATLAB as shown in the Fig. 3.

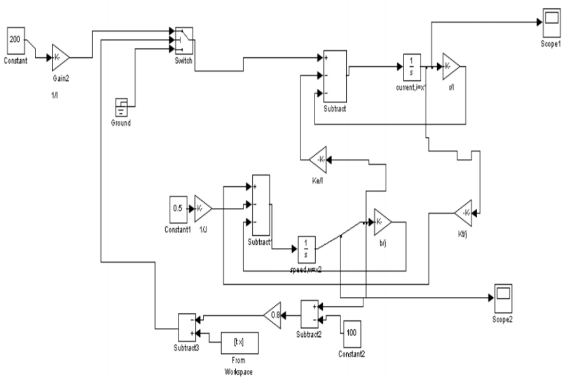

Fig. 3: Equations of the drive simulated with SIMULINK

The dynamic behavior of the DC drive can be found from the bifurcation diagram drawn with different parameters value. The bifurcation diagram with input voltage as the parameter is shown in Fig. 4. It is essential to study the occurrence of chaos due to the variation of system parameters. When bifurcation occurs, an abrupt change in the steady state behavior of the system also occurs. A bifurcation diagram represents all plots of the steady state orbits as a function of a bifurcation parameter.

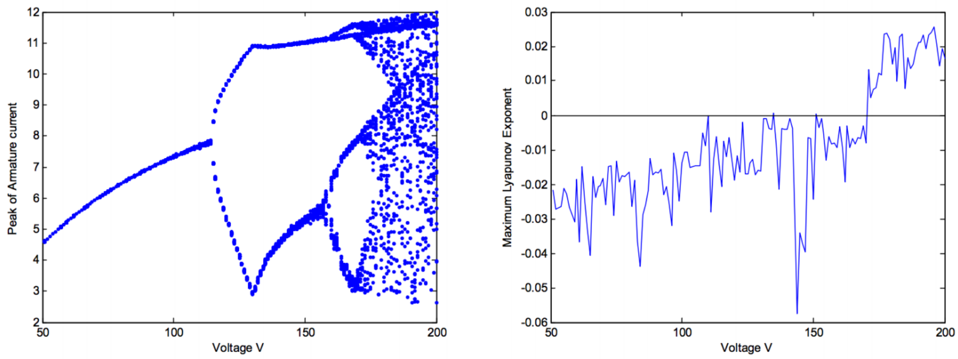

Fig. 4: Bifurcation diagram with input voltage as parameter and plot of maximum Lyapunov exponent

This shows that a period doubling route to chaos is followed. The occurrence of chaos is quantified by maximum Lyapunov exponent.

The Period-1 behavior is observed up to input voltage, 120V. Fig. 5 shows the simulated waveform of armature current for period-1 orbit at input voltage 100 V. Fig. 6 shows the time plot of speed and phase plot of armature current and speed for the same voltage. Bifurcation to different periodic orbit and chaos is observed with the increase of input voltage. The system bifurcates to period-2 after period-1. After period-2, period-4 behavior starts at a voltage of 158 V and continues up to 170 V. Period-8 behavior is observed after that. The period-8 converges to chaos at 170.5 V.

The period-2 sub harmonic waveform is shown in Fig.7. The period-2 behavior changes to period-4 after that. The period-4 behavior that occurs at 160V is shown in the Fig. 8. At input voltage of 170.5, the system shows period-8T sub harmonic (Fig.9, 10). Finally chaotic behavior of the drive is shown in Fig. 11.

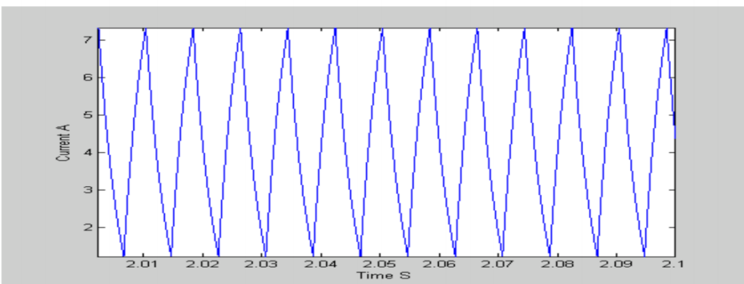

Fig. 5: Time plot of armature current showing P-1 at input voltage 100 V

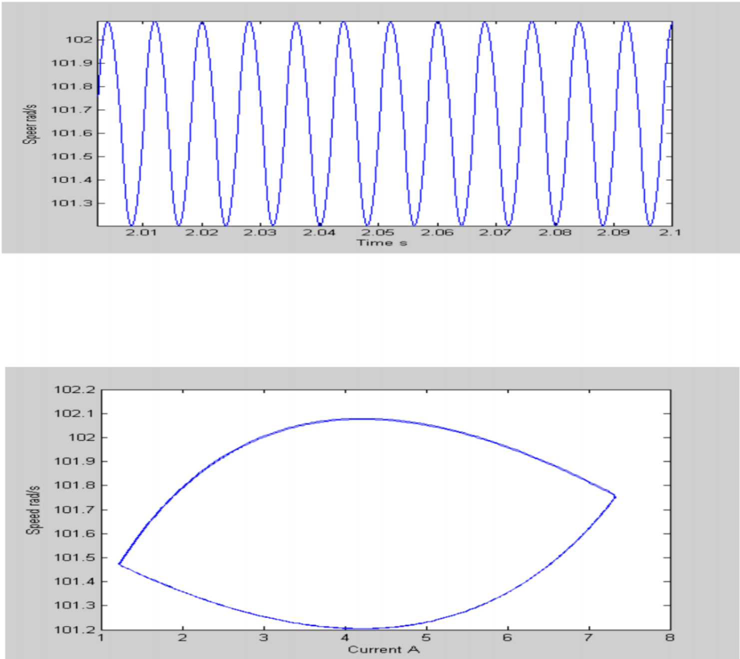

Fig. 6: (*Upper*) Time plot of speed and (*Lower*) Phase plot of armature current and speed showing P-1 at input voltage 100V

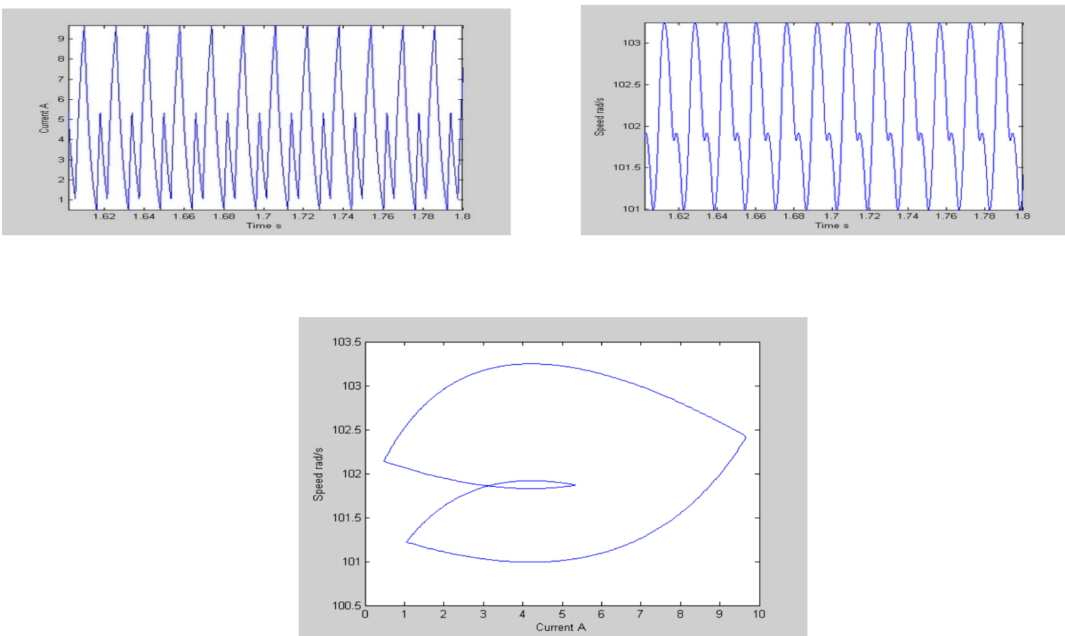

Fig. 7: Time plot of armature current, speed and phase plot showing P-2 at input voltage 150V

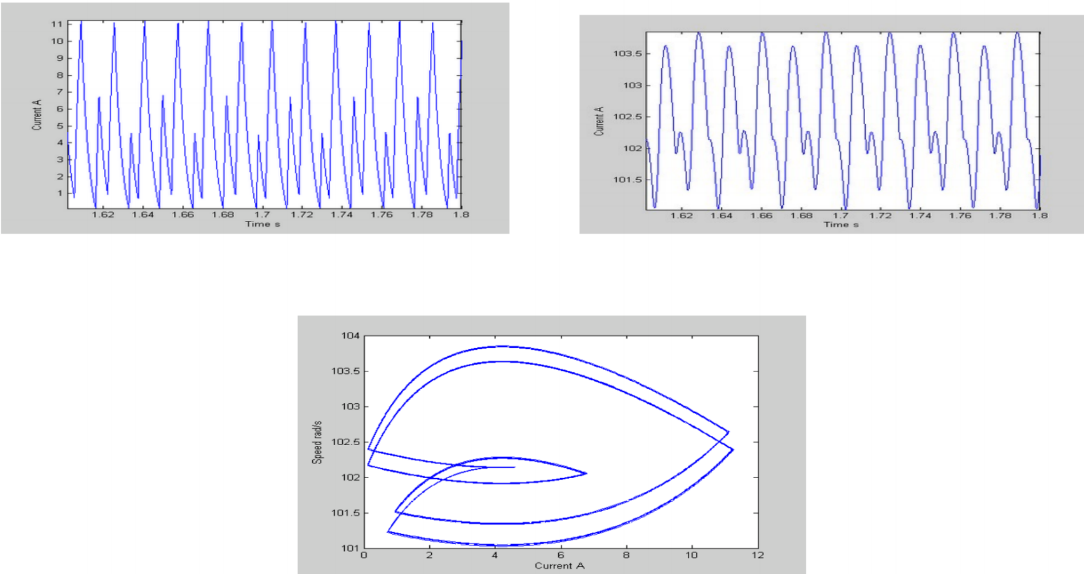

Fig. 8: Time plot of current, speed & phase plot showing P-4 at input voltage 160 V

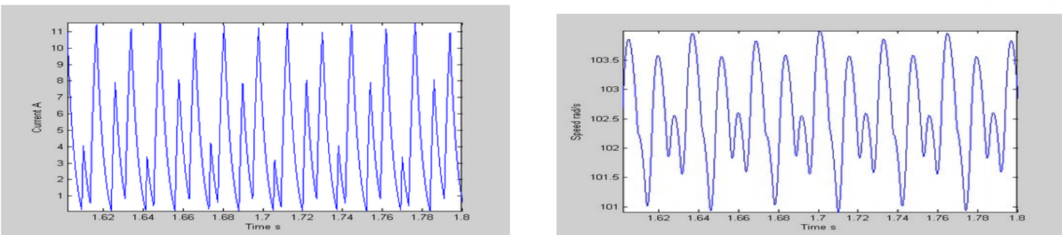

Fig. 9: Time plot of current & speed showing P-8 at input voltage 170.5 V

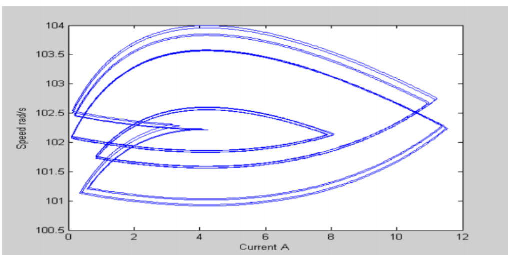

Fig. 10: Phase plot showing P-8 at input voltage 170.5 V

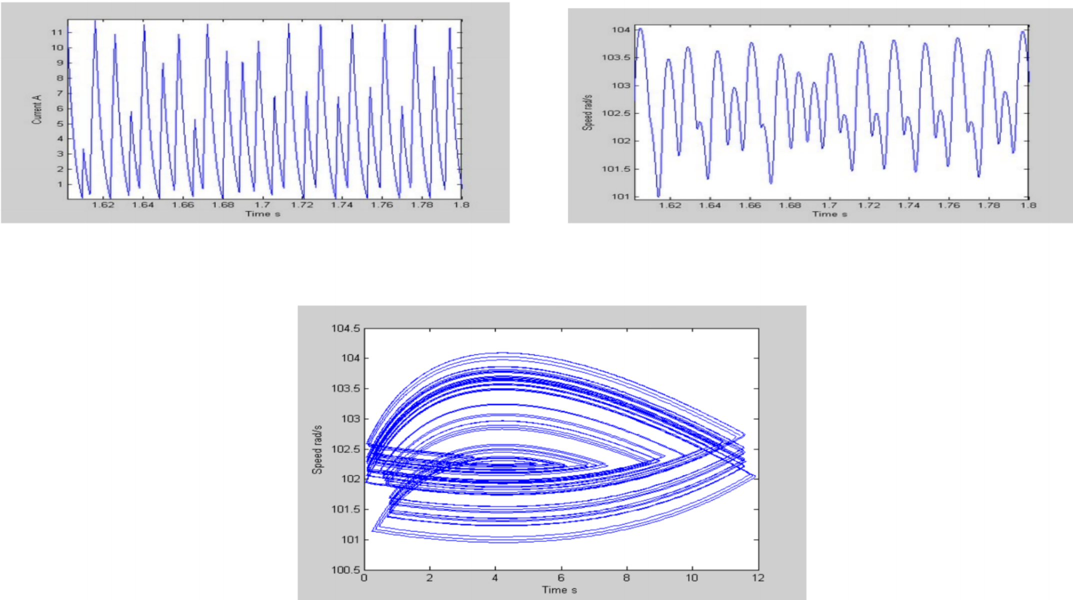

Fig. 11: Time plot of current, speed and phase plot showing chaos at input voltage 200 V

Fig. 12 shows the bifurcation diagram with gain as the parameter. The period-1 behavior is observed up to gain 0.32. The period-1 bifurcates to period-2 at 0.33. The period-2 continues up to gain 0.43. Then period-4 starts. After that the period doubling route to chaos is followed There are periodic windows in the chaotic zone. Period-1 behavior is observed at gain of 1.74. The evidence of periodic windows in the chaotic zone is evident from the plot of Lyapunov exponent. For periodic orbit Lyapunov exponent is negative and it is positive for chaos.

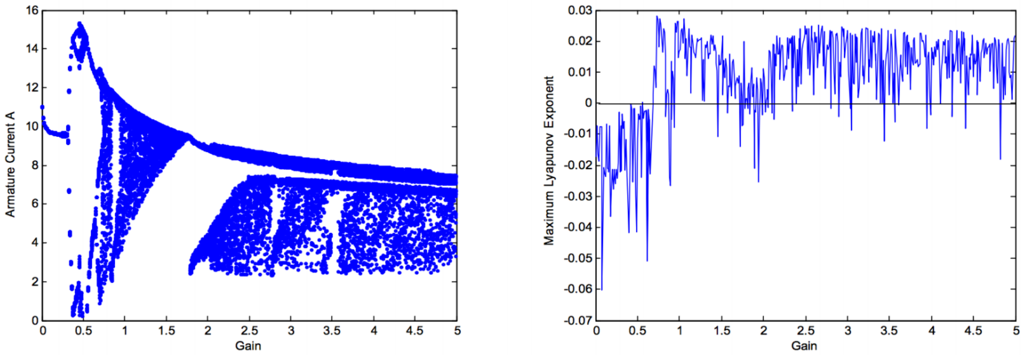

Fig.12: Bifurcation diagram with gain as parameter & plot of maximum Lyapunov exponent.

The bifurcation behavior with load torque as parameter is shown in Fig. 13. This behavior is the reverse of behaviors shown in Fig. 4 and Fig. 12. The system shows chaotic behavior at lower value of torque. With decrease of value of torque, period 1 bifurcates to period 2 and then to chaos.

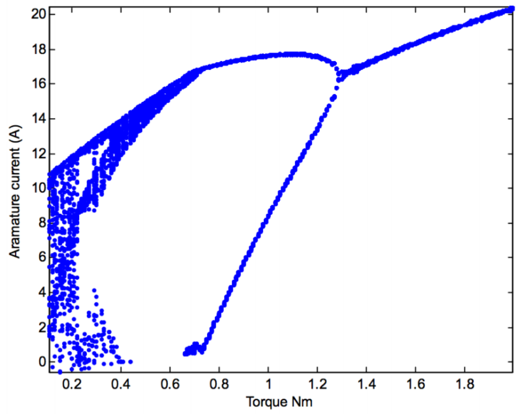

Fig.13: Bifurcation diagram with load torque as parameter

4. Stability analysis of Period-1 Orbit

We are interested in the stability of period-1 orbit of the DC drive for its normal operation. This orbit starts at a clock instant and comes back to the same state at the end of the clock period. The DC drive is not a smooth system. The equation of evolution during switch on and switch off and vice versa are smooth as shown in equation (2&3). However, at the instant of switching, the evolution becomes nonsmooth. Filippov [8] showed that in such a situation one has to additionally consider the evolution of the system across the switching event. It is known that in linear time invariant (LTI) system, the state transition matrix is given by the $\phi = e^{At}$ where A is the state matrix. The basis of the Filippov's method is to find out state transition matrix over the complete cycle including switching. The whole cycle include the switch on duration, switch off duration and switching instants. The state transition matrix that relates the evolution just after the switching event to that just before is called the saltation matrix, S and is given by

$$S = I + \frac{(f_{on} - f_{off})n^T}{n^T f_{off} + \frac{dh}{dt}}$$

where I is the identity matrix, ***h(x,t)=0*** represents the switching condition, ***n*** is the vector normal to the switching surface and $\boldsymbol{n^T}$ is its transpose, $\boldsymbol{f_{off}}$ represents the right hand side of the differential equation before switching occurred and $\mathbf{f_{on}}$ represents the right side of the differential equation after switching. From this the saltation matrix is calculated after each switching event. Since we are interested in the stability of the periodic orbit exhibited by the drive, we need to calculate the state transition matrix over the whole cycle. This matrix is called monodromy matrix. If the switch is ON at time **0** till time ***dT*** (where ***d*** is the duty cycle and ***T*** is the period) and the switch remain OFF from ***dT*** till time **T**, then the monodromy matrix is expressed as

$$\phi_{cycle}(T,0) = S_2 \times \phi_{off}(T,dT) \times S_1 \times \phi_{on}(dT,0)$$

where S_1 is the saltation matrix related to first switching and S_2 is that related to the second switching. If the moduli of all the eigen values of the monodromy matrix, called Floquet multiplier, are less than unity, then the system will be stable.

For the drive under consideration

$$f_{on} = \begin{bmatrix} -\dfrac{Ra}{La}x(1)_{off} - \dfrac{K_E}{L}x(2)_{off} + \dfrac{V}{La} \\ \dfrac{K_T}{J}x(1)_{off} - \dfrac{B}{J}x(2)_{off} - \dfrac{T_L}{J} \end{bmatrix}$$

$$f_{off} = \begin{bmatrix} -\dfrac{Ra}{La}x(1)_{off} - \dfrac{K_E}{La}x(2)_{off} \\ \dfrac{K_T}{J}x(1)_{off} - \dfrac{B}{J}x(2)_{off} - \dfrac{T_L}{J} \end{bmatrix}$$

$x(1)_{off}$ **and** ***$x(2)_{off}$*** **represents the values of state variables (current and speed) at the instant of switch on.**

$$\phi_{on} = e^{AdT}, \phi_{off} = e^{A(1-d)T}$$

$$h(x,t) = G[x(2) - \omega_{ref}] - V_L - \frac{(V_u - V_L)t}{T} = 0 \qquad n = \begin{bmatrix} \dfrac{dh}{dx(1)} \\ \dfrac{dh}{dx(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ G \end{bmatrix} \; \& \; \frac{dh}{dt} = -\frac{V_u - V_L}{T}$$

$$f_{on} - f_{off} = \begin{bmatrix} \dfrac{V}{La} \\ 0 \end{bmatrix} \; \& \; (f_{on} - f_{off})n^T = \begin{bmatrix} 0 & \dfrac{GV}{La} \\ 0 & 0 \end{bmatrix} \quad n^T f_{off} = G \times \frac{K_t x(1)_{off} - Bx(2)_{off} - T_l}{J}$$

Consequently, the saltation matrix for the DC drive at the switching instant (1-d) T is given by

$$S = \begin{bmatrix} 1 & \left(\dfrac{\dfrac{GV}{La}}{G\left(\dfrac{K_t x(1)_{off} - Bx(2)_{off} - T_l}{J} \right) - \dfrac{V_u - V_l}{T}} \right) \\ 0 & 1 \end{bmatrix}$$

For the given set of parameter and input voltage 100V, we obtain d= 0.4706 and

$$\phi_{on} = \begin{bmatrix} 0.3734 & -0.0108 \\ 0.3264 & 0.9942 \end{bmatrix}, \phi_{off} = \begin{bmatrix} 0.3298 & -0.01116 \\ 0.3488 & 0.9932 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 1 & -4.4283 \\ 0 & 1 \end{bmatrix}, \phi_{cycle} = m = \begin{bmatrix} -0.4574 & -1.6573 \\ -0.0498 & -0.4521 \end{bmatrix}$$

The second saltation matrix $\mathbf{S_2}$ is related the switching from the ON state to the OFF state at the end of the clock cycle turns out to be identity matrix as the falling edge of the ramp causes the **dh/dt** term to be infinity.

The values of ***x (1)*** $_{off}$ and **x (2)** $_{off}$ can be found out by solving state equations (3). The duty cycle, **d**, is found out by solving the equation ***h(x,t)=0.***

The eigen values of the monodromy matrix were found to be -0.7420 and -0.1675. This is expected from the bifurcation diagram (Fig. 4). All eigen values are less than unity. Hence the period-1 orbit is stable.

For some values of the supply voltage $\in (55,120)$ and gain, $\in (0.1, 0.4)$ the Floquet multipliers were computed and the results are shown in Table 1 and Table 2. The corresponding loci are shown in Fig. 14 & 15. A bifurcation is described by crossing of multiplier from the interior of the unit circle to the exterior.

Table 1:

Voltage	Floquet multipliers	Remark
55	0.2910 $\pm$j0.0.19901	Period-1 stable
100	-0.7432,-0.1672	Period-1 stable
116	-0.9209,-0.1349	Period-1 stable
120	-1.0661,-0.1165	Period -2

Table 2:

Gain	Floquet multipliers	Remark
0.1	0.0898 $\pm$j0.3409	Period-1 stable
0.2	0.2542 $\pm$j0.2442	Period-1 stable
0.32	-0.8710,-0.1427	Period-1 stable
0.38	-0.1130,-1.0992	Period -2

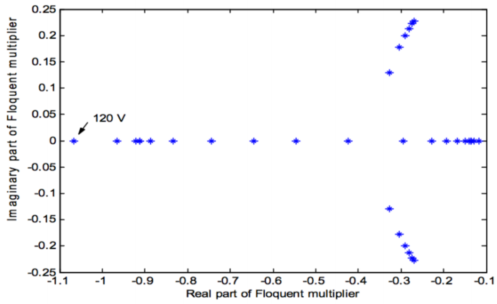

Fig. 14: Loci of Floquet multipliers for V $\in$ (55,120)

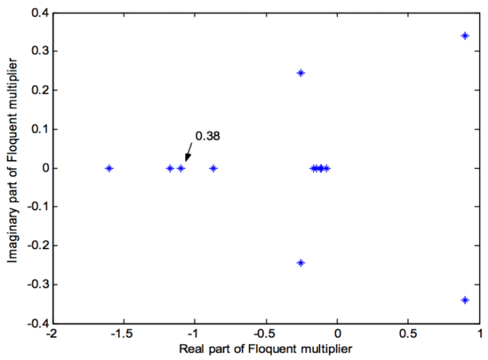

Fig.15: Loci of Floquet multipliers for gain ∈ (0.1,0.4)

5. Conclusion

In this paper occurrence of chaos has been demonstrated by MATLAB simulation. The bifurcation diagram drawn gives the behavior of the system at different parameters. The calculation maximum Lyapunov exponent quantifies the occurrence of chaos. The stability analysis with the help of monodromy matrix and its eigen values gives the stability properties of the drive. The stability of period-1 can be greatly influenced by appropriate manipulation of the saltation matrix of the system. Based on the observation, we are able to design a controller capable of extending the parameter range for stable period-1 operation. This information is very important for the designer to design the drive system.

Acknowdgement

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