



## Irradiation Enhanced Synthesis and Characterization of Biodiesel from Vegetable Oil-A Short Review

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**Abstract:** Depletion of petroleum resources continues to be an important global issue. To overcome this issue, biodiesel is gaining more and more importance as an alternative, renewable and sustainable fuel. Chemically biodiesel is ester of monoalkyl fatty acid derived through catalytic transesterification process from mostly non-edible vegetable oil. The transesterification process is influenced by the mode of reaction condition, molar ratio of alcohol to oil, type of alcohol, reaction time and temperature. Transesterification process by conventional heating needs more reaction time which is dramatically reduced by using suitable irradiation techniques like ultrasound, microwave or sunlight as heating sources. Other parameters like alcohol-oil ratio and quantity of catalysts are also influenced by the irradiation. Choice of catalyst along with the type of irradiation is also a critical parameter. Type and power of irradiation also influence some of the process parameters. In this paper, available literature on irradiation-enhanced synthesis of biodiesel with different combination of oil and catalysts has been reviewed.

**Keywords:** Biodiesel; Vegetable oil; Monoalkyl Fatty acid ester; Transesterification; Light; Ultrasound; Microwave

### 1. Introduction

With the advancement of industrialized civilization, conventional fuels are getting scarce and costly. Moreover emissions from their combustion pollute the environment. Alternative and non-polluting fuels are therefore, demand of the day. Hydrogen, natural gas or biofuels are among such alternative fuels that have high potential for mass consumption.

Biofuels are organic combustible matter that are derived from biomass; biofuels can be solid (wood or cow dung cake), liquid (alcohols) or gas (pyrolysis gas). The advantage of biofuel is that, they recycle carbon dioxide without increasing the net quantity of the same in the atmosphere, if their consumption is proportionate with their generation. For internal combustion engines, liquid biofuels can be of two types – bioalcohols and transformed oils. However, biofuel and biodiesel are not synonymous- according to a few scientists, biofuels can be of three generations. The first generation of biofuels includes edible oil used both for alcohol and biodiesel production, the second generation comprises biofuels and biodiesel derived from non-edible plant-products and the third generation includes biodiesels produced by the microalgae.

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According to the American Society for Testing and Materials Standard biodiesels are defined as '*monoalkyl esters of long chain fatty acids derived from renewable lipids such as vegetable oils or animal fats used in ignition engines*' [1].

Due to this similar structure as fossil fuel, biodiesel is accepted as a suitable substitute and no modification is required for the engine. The major advantages of biodiesel over the conventional diesel oil are its biodegradability, non-toxicity, low emission profile, easy availability of raw materials and similar chemical structure. Biodiesel is used either in the pure form or as a blend with fossil-diesel.

The history of exploring biodiesel is more than 100 years old. Rudolf Diesel tested the performance of an engine with peanut oil in 1900 in Paris [2]. But due to the low volatility, high viscosity, high flash point and residue after combustion they could not be considered as an engine-fuel. Hence modification of vegetable oils were necessary to make it suitable for injection in the engines were necessary. Transesterification is one such modification [3].

The technology usually employed to produce biodiesel, is transesterification of the vegetable oil. Many researches have shown that biodiesel production from non-edible vegetable oil is more economic than edible oil source [4]. Transesterification is the process of converting a vegetable oil into an ester using an alcohol (e.g. methanol, ethanol), in presence of a catalyst like sodium hydroxide or potassium hydroxide with glycerol as a byproduct [5]. In the conventional method, the necessary energy is provided by heating accompanied by mechanical mixing. It needs much more reaction time of more than an hour. Higher energy consumption, high equipment cost, usage of excess amount of alcohol and longer reaction time are the drawbacks of conventional method. Theoretically, each mole of biodiesel is generated out of 1 mole of alcohol and 1/3 mole of triglyceride. However, an excess of alcohol is usually added to increase the yield of biodiesel [6].

This reaction time can be minimized from one hour to five minutes by using different irradiations as sources of energy [7]. Irradiation energy interacts with the reactants at a molecular level, thus very efficient and rapid heating can be obtained. Since the energy is interacting with the molecules at a very fast rate, the molecules do not have time to relax and heat is generated at short times results speedy completion of reaction.

## **2. Oils generally used for biodiesel**

It is difficult to get raw material (vegetable oil) for production of biodiesel. If the mono-unsaturated fatty acid content is high, kinematic viscosity, density and cold flow properties may improve but properties like cetane number or calorific value may deteriorate. On the other hand, with increasing chain length, the calorific value and cetane number increases, but some other properties like oxygen content, kinematic viscosity, density and cold properties are negatively affected [4].

A few oils used for production of biodiesel are rapeseed, soyabean, coconut, palm [8], sunflower [9], corn, ricebran [10], castor [11], karanja [12], tobacco seed [13]



and *Jatropha* [14]. Sometimes waste cooking oils are also used for synthesis of biodiesel [15 -- 17].

### 3. Alcohols used for biodiesel products

The most used alcohol for biodiesel production is methanol; it does not form azeotropic mixture with water and hence easy to separate after use. Ethanol and isopropanol may also be used, but ethanol forms an azeotropic mixture with water. A critical factor is the water content. Water interferes with transesterification reactions when using alkaline catalyst and can result in poor yields of BD, high levels of soap, FFA and triglycerides [18].

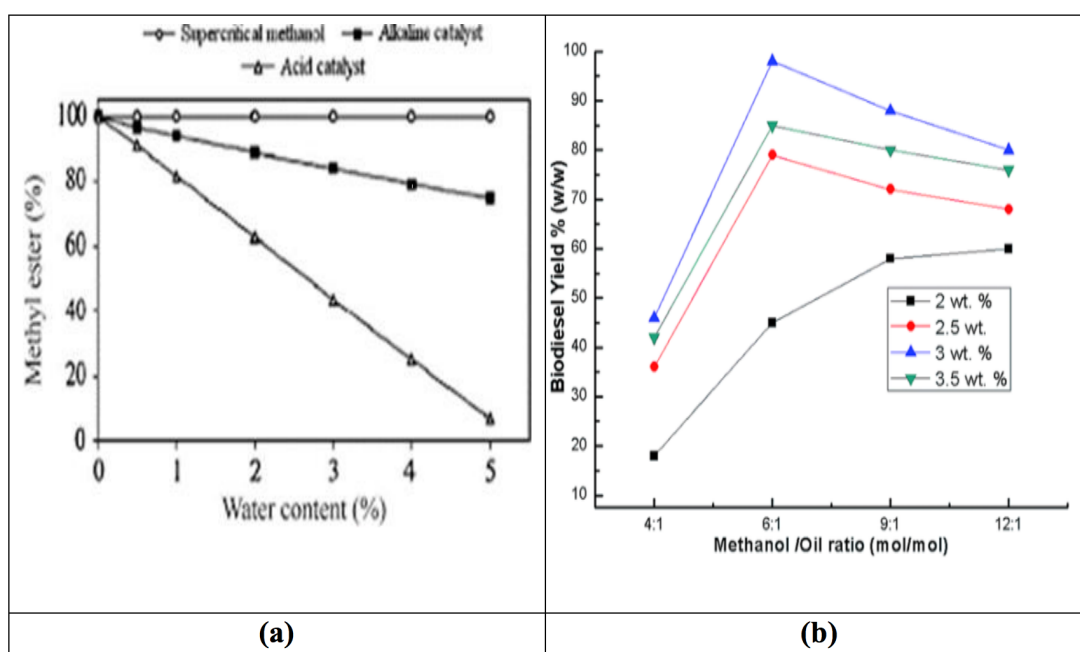


Fig. 1: (a) Influence of water content on yield of biodiesel [19]; (b) Influence of methanol: oil ratio on yield of biodiesel [20]

Alcohol to oil molar ratio is an important parameter in biodiesel production. As this is a reversible reaction, one can increase the molar ratio of alcohol to obtain more biodiesel. Also if excess alcohol is added to the oil, more undesirable product glycerol is produced. Moreover it can dilute the catalyst concentration. Many researchers have shown that 6:1 alcohol to oil molar ratio leads to maximum production rate [21].

### 4. Transesterification mechanism

Synthesis of biodiesel through transesterification reaction refers to a catalytic chemical reaction involving vegetable oil and alcohol to yield fatty acid alkyl esters (biodiesel) and glycerol (by product) [22 – 26].

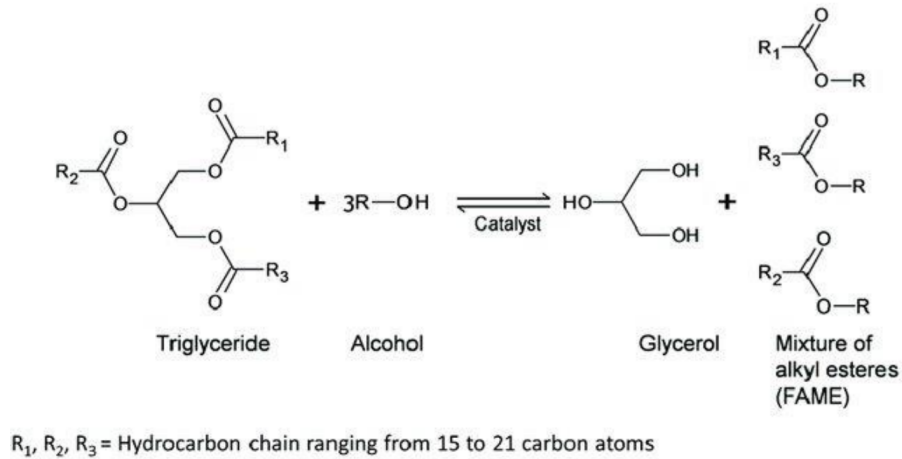


Fig. 2: Schematics of overall transesterification reaction [27]

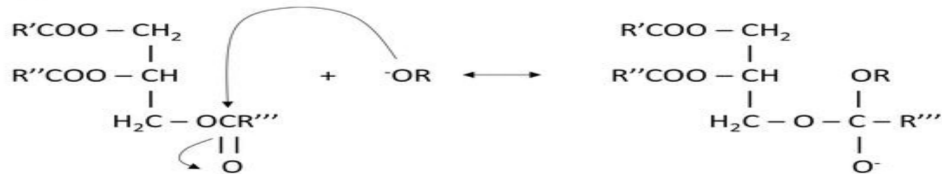
Transesterification is the reaction of an oil or fat with an alcohol to form esters and glycerol. There are three reaction steps involved in the transesterification process of triglycerides [28]. In the first step of reaction, triglyceride molecule reacts with an alcohol molecule (mainly used methanol) to produce diglyceride and a fatty acid alkyl ester. Then in the second reaction, diglyceride reacts with alcohol to form monoglyceride and another molecule of fatty acid ester. In the third reaction, monoglyceride reacts with alcohol to produce glycerol and a third molecule of fatty acid ester.

Transesterification is either catalytic (acid or base) or non-catalytic (using supercritical fluid) [29 – 33].

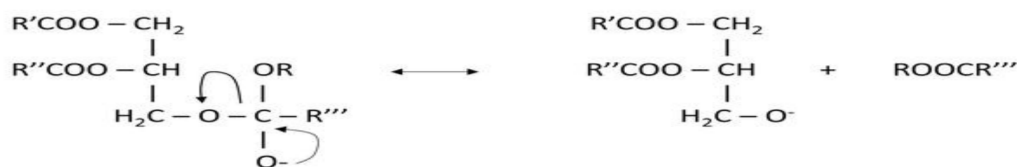
Step 1:



Step 2:



Step 3:



Step 4:

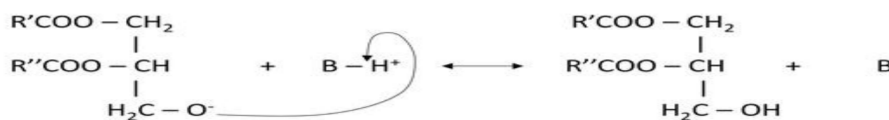


Fig. 3: Mechanism of base catalyzed transesterification [34]

For a base-catalyzed reaction, first a reaction of base with alcohol occurs to form alkoxide with the protonated catalyst. Then nucleophilic attack takes place at the carbonyl carbon of triglyceride molecule by alkoxide ion to form a tetrahedral intermediate. This intermediate is rearranged to give rise to alkyl ester and a diglyceride anion. Diglyceride anion deprotonates the catalyst to form active catalyst and diglyceride. Thus diglyceride and monoglyceride are transformed into alkyl ester (FAME) and glycerol using the same mechanism [35 – 38].

In case of an acid-catalyzed reaction, carbonyl oxygen is protonated by  $H^+$  which results in cation formation. Then nucleophilic attack of alcohol occurs to produce tetrahedral intermediate. This intermediate is rearranged to release an alkyl ester (FAME) and proton catalyst [39 – 41].

For oils having high amount of free fatty acids, a two-step process, using two types of catalysts, is recommended [42]. First acid catalyst is employed for esterification of free fatty acid to ester. When free fatty acid level reaches less than 0.5-1%, then base catalyst is employed for transesterification step. But the major drawback of this process is the requirement of extra separation stages, washing and catalyst removal in both steps [43, 44].

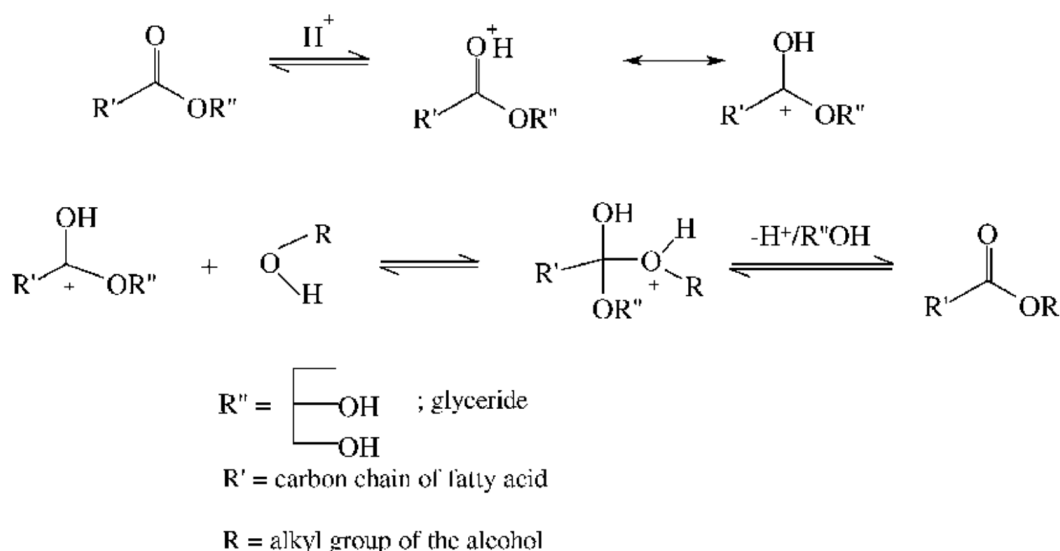


Fig. 4: Mechanism of acid catalyzed transesterification [45]

There may also be a few competitive side reactions. If water is present, especially at a high temperature, it may hydrolyze the triglyceride to diglyceride and free fatty acid. Another undesired side reaction is production of soap by the reaction of free fatty acid and alkaline catalyst [46].

## 5. Catalysts used for synthesis of biodiesel

The catalysts in the conventional transesterification reaction may be homogeneous or heterogeneous. The most common homogeneous catalysts are alkaline sodium hydroxide and potassium hydroxide. Acids may also be used as catalysts for oils with high fatty acids, but the rate of reaction is too low for industrial production and acids



also need a neutralization after the reaction is over [47]. Alkaline catalysts are less corrosive than acids and gives faster reaction rate. Another drawback of acid catalyzed process is catalyst separation after the reaction is over. So alkaline catalyzed transesterification is more preferred than acid catalyzed processes [48].

Among the heterogeneous catalysts, alkaline earth oxides, alkali doped materials, transition metal oxides, hydrotalcites, mesoporous silicas, hetero-polyacids, acidic polymers and resins, waste carbon-derived solid acids or miscellaneous solid acids are included. Transition metal oxides like  $\text{TiO}_2$  or  $\text{ZnO}$  act as photocatalysts under UV or sunlight in light-enhanced transesterification process [47].

## 6. General characteristics and standards of biodiesel

The American Society for Testing and Materials (ASTM) standard D6751 and the Committee of Standardization in Europe (CEN) standard EN 14214 in Europe stipulate several quality specifications that must be met before biodiesel can be used as a direct fuel replacement or as blend with petroleum [49 – 52]. A summary of the standards of the ASTM and EN are given in Table-1 below:

Table 1: ASTM and EN standards for biodiesel [ASTM D6751 and EN 14214]

Property	Unit	Limits		Test Method	
		ASTM D6751	EN 14214	ASTM D6751	EN 14214
Viscosity at 40°C	mm <sup>2</sup> /s	1.9-6.0	3.5-5.0	D445	EN ISO 3104
Flash point	°C	130	101	D93	ISO CD 3679e
Density at 15°C	kg/m <sup>3</sup>	-	860-900	-	EN SIO 3675
Cetane number	-	47	51	D613	EN ISO 5165
Water content	mg/kg	-	500 max	-	EN ISO 12937
Methanol content	%(m/m)	-	0.2 max	-	pr EN 141101
Acid value	mg KOH/g	0.80 max	0.5 max	D664	pr EN 14101
Free glycerol	%(m/m)	0.020max	-	D6584	pr EN 14106

In India, IS-15607-2016 describes the standard for Fatty acid methyl ester (FAME). However the quality of blend biodiesel was governed by IS-15607-2005. In Table-2 a few parameters as stipulated in both the standards are compared.

Table 2: Comparison of Indian standards for blend biodiesel and FAME [IS-15607-2005 and 2016]

Property	Unit	Limits		Test Method	
		IS-15607-2005	IS-15607-2016	IS-15607-2005	IS-15607-2016
Viscosity at 40°C	mm <sup>2</sup> /s	2.5- 6.0	3.5-5.0	ISO 3104	ISO 3104
Flash point	°C	120	101	-	ISO 2719
Density at 15°C	kg/m <sup>3</sup>	860-900	860-900	ISO 3675	ISO 3675/ISO 12185/ D 4052
Cetane number	-	51	51	ISO 5156	ISO 5165
Water content	mg/kg	500	500	D 2709	ISO 3733/ ISO 6296/ ISO 12937
Methanol content	%(m/m)	0.2	0.2	EN 14110	EN 14110
Acid value	mg KOH/g	0.50	0.50	-	EN 14104/ASTM D 974
Free glycerol	%(m/m)	0.02	0.02	D 6584	D 6584/EN 14105/ EN 14106

## 7. Irradiation-enhanced synthesis of biodiesel using different irradiation sources

A group of irradiation-enhanced technology has been introduced for synthesis of biodiesel including photocatalysis, ultrasonication and microwave irradiation. The purpose of the irradiation is to provide more intimate contact and excitation of the reactant. A few advantages are there for the irradiation-enhanced synthesis of biodiesels over the conventional process - in most cases, the reaction conditions are moderate, the closer contact between reactants leads to a significant reduction of both catalyst and/or alcohol and higher biodiesel yield in less reaction time is reached [48].

Aim of this review is to compile the three above-mentioned techniques for the irradiation-enhanced synthesis of biodiesel and also to compare the processes as well as the quality of biodiesel obtained by such methods.

## 8. Ultrasound-enhanced synthesis of biodiesel

Sonochemistry refers to the application of the ultrasound with a frequency greater than the upper limit of human hearing (~20 kHz to 10 MHz) into chemical reaction.

Ultrasound energy is a useful tool to enhance both the mass transfer and chemical reactions resulting in shorter reaction time, cheaper reagents and less extreme physical conditions. Ultrasound wave alternately compresses and stretches the molecular spacing of the medium during the chemical process and easily can make fine emulsions from immiscible liquids. By introducing high power ultrasound into a liquid medium, the sound waves are transmitted in the fluid and create alternating high pressure (compression) and low pressure (rarefaction) cycles, with rates depending on the frequency. During the low pressure cycle, high intensity ultrasonic waves create small vacuum bubbles or voids in the liquid. Acoustic cavitation is the growth and collapse of cavitation bubbles, which when exploded, generates an increase in temperature in the reaction medium. This phenomenon is able to increase the speed of the reaction and reduce the reaction time [53]. Among the irradiation enhanced processes for synthesizing biodiesel ultrasonic process is the most suitable for large scale production [48].

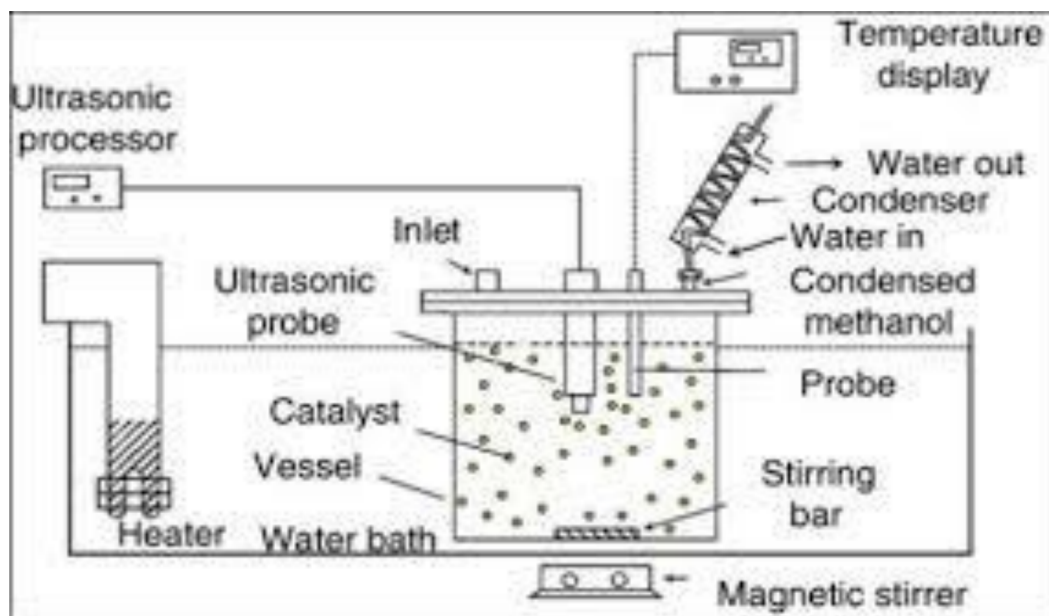


Fig. 5: Ultrasound enhanced synthesis of biodiesel - a schematic diagram for the Set-up [54]

There may be three types of devices for introducing ultrasound in the reaction mixture – ultrasonic bath, ultrasonic probe or horn and sonochemical reactor. Bath type sonicator passes ultrasound indirectly through the water taken in the bath to the solution whereas probe type sonicator inserts ultrasound directly to the reaction mixture [55]. Hence in bath type sonicator, the distribution of sound energy may not be uniform and this is a limitation of the bath type sonicators.



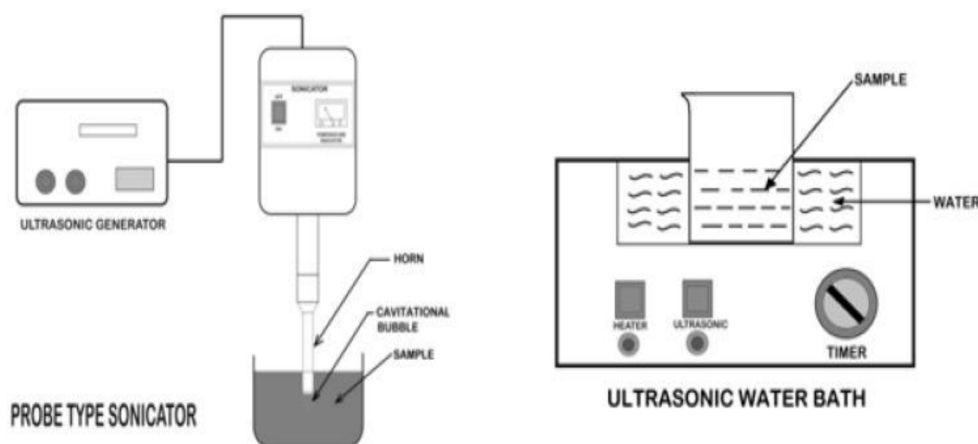


Fig. 6: Arrangements of probe and bath type sonicators [55]

In the study by Georgogianni et al. [56] an ultrasonic probe (24 kHz, 200W) was used in the transesterification reaction of sunflower oil with methanol. After 20 min of reaction, a conversion of 95% was obtained at alcohol to oil molar ratio of 7:1 in the presence of 2% of the sodium hydroxide catalyst. The same authors obtained 96% yield in 40-60 minutes with rapeseed oil using a bath type ultrasonicator of same power [57]. Fan et al. [58] also performed biodiesel production by using ultrasonic probe (40kHz, 200W). They used cotton seed oil and methanol at alcohol to oil molar ratio of 6:1 and 1% potassium hydroxide. In 5 min of reaction, it reached 96% biodiesel yield. A 99% biodiesel yield was obtained by Santos et al. [59] by using ultrasonic probe (40kHz,200W) at methanol to waste cooking oil molar ratio of 9:1 and 3% sulfuric acid catalyst. Babajide et al. [60] obtained 96.8% yield with waste cooking (sunflower and soyabean) oil, 6:1 methanol and 0.75% KOH in 20 minutes using 24kHz probe type sonicator. Hyyan et al. [61] obtained 96.3% yield with methanol (7.5:1) using 0.7%  $\text{H}_2\text{SO}_4$  catalyst in 300 minutes using 40 kHz bath type sonicator. Kumar et al. [62] developed biodiesel from coconut oil catalyzed by potassium hydroxide. An ultrasonic probe was used at 24kHz, 200W. After 7 min of reaction, a maximum conversion of 98% was obtained with an ethanol to oil molar ratio of 6:1, 0.75% catalyst concentration. Rapeseed oil at 45°C was used by Xuan et al. [63] with 0.5% methanol and 4W sonication through a probe to get a yield of 80% in 20 minutes. Deshmane et al. [64] produced biodiesel using an ultrasonic bath with frequency of 25kHz. In this study, the highest conversion 80% was obtained at isopropanol to palm oil molar ratio of 5:1 and 5% catalyst concentration. Thanh et al. [65] also produced 99% biodiesel yield from canola oil by using ultrasonication (20 kHz) at methanol to oil molar ratio of 5:1 and 0.7% sodium hydroxide catalyst. 98% biodiesel yield was achieved by Santos et al. [66] from *oreochromisniloticus* oil catalyzed by sulfuric acid. The reaction was carried out under ultrasonication (40kHz) at methanol to oil molar ratio of 9:1.

A few process parameters are influenced by the ultrasound. Reaction time is reduced from the order of hours in conventional process to the order of minutes by using ultrasound. Due to an efficient dispersion, efficiency of catalysts, especially that of heterogeneous catalysts are increased; required quantity of the catalyst is also reduced. In conventional transesterification, energy is required both for agitation and for heating the reaction mixture. Since ultrasound mixes and heats up the reaction mixture at a time, some energy is saved. Alcohol: Oil ratio is an important parameter – by using ultrasound, it is observed that less quantity of alcohol is required compared to the conventional process for same yield of biodiesel [1].

## 9. Microwave-enhanced synthesis of biodiesel

Among the sources of irradiation, microwave is an important one [67, 68]. Biodiesel can also be produced using microwaves. Microwaves are part of electromagnetic waves between the radio frequency and inferred frequency ranges (2.45GHz) [69, 70]. Conventional heating process in biodiesel production requires large amount of heats and the temperature is applied to the material surface only whereas microwave systems can directly transfer thermal energy into interior molecules of the reactants [71]. Controlled and accurate heating, short reaction time, quality and property improvement are the keys to choose microwave technique over the conventional heating process [72].

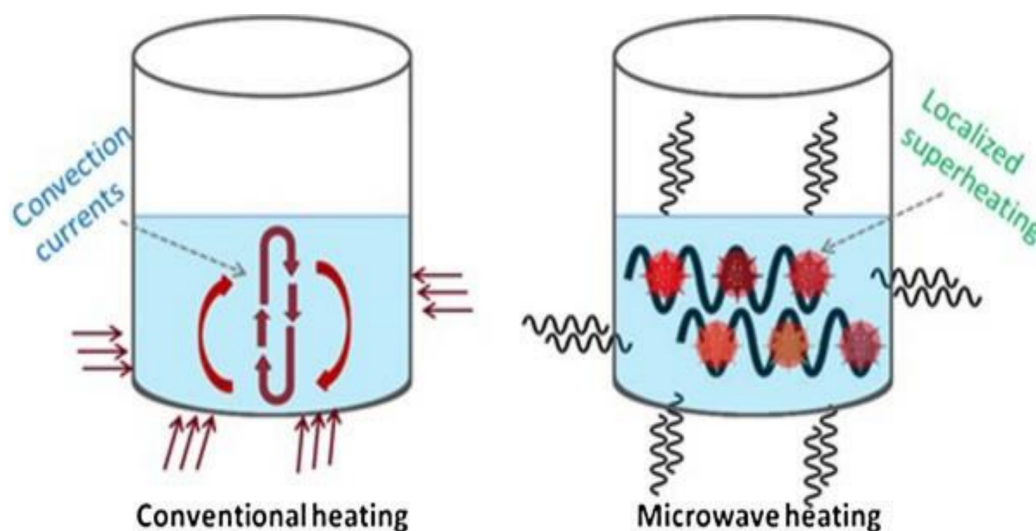


Fig. 7: Microwave enhanced synthesis of biodiesel - a schematic set-up [73]

Microwave irradiation rises the temperature of the whole volume evenly and simultaneously whereas in conventional heating the reaction mixture in contact with the vessel wall is heated first. If a molecule possesses a dipole moment, when it is exposed to microwave irradiation, the dipole tries to align with the applied electric field. Since the electric field is oscillating, the dipoles constantly try to realign to follow this movement. This continual reorientation of the molecules results in friction and thus heat. It needs less reaction time [74].

One major issue of using microwaves is the high cost of the reactors – researchers modified domestic microwave ovens into laboratory scale reactors to overcome such problem [75].

Patil et al. [38] produced 92% biodiesel yield from waste cooking oil by using microwave irradiation at methanol to oil molar ratio of 9:1 and 2% potassium hydroxide catalyst. 98.85% biodiesel yield was reported by Azcan et al. [76] at 6:1 methanol to waste frying oil molar ratio catalyzed by 1.5% sodium methoxide under microwave irradiation. Hincapé et al. [77] also performed biodiesel production by using microwave irradiation. They used castor oil and ethanol at alcohol to oil molar ratio of 10:1 and 1.5% potassium hydroxide. Li et al. [78] observed 93.51% biodiesel from soybean oil catalyzed by sodium potassium tartrate doped zirconia under microwave irradiation.

Process parameters that can influence the microwave-enhanced production of biodiesel are catalyst type and quantity, reaction time, microwave power, fatty acid and water content, reaction temperature and alcohol to oil ratio.

It was observed that an increase in microwave power generally increase the percent yield upto a critical value, after that the yield decreases due to damage the molecular structure of triglyceride at high power microwave irradiation Groisman and Gedanken [79]. Sherbiny et al. [74] obtained 83% yield of biodiesel from Jatropa oil in 30 minutes using 1% KOH and 1200 W power of microwave. However Lin and Chen [80] achieved 90% yield with the same raw material and catalyst using 950W microwave in 10 seconds.

The same trend is also found in case of irradiation time; increase in irradiation time increases the yield upto a critical value. Castor oil with 5%  $\text{H}_2\text{SO}_4$  catalyst and 200W microwave power yielded 43% biodiesel in 10 minutes, 54% in 20 minutes and 93% in 180 minutes [81].

Regarding heterogeneous catalysts, performance of CaO is good with microwave. Particlesize of the solid catalyst is also a critical factor. Khemthong et al. [82] used CaO powder with palm oil (olein) under 900W microwave. They obtained 96.7% yield in only 4 minutes. Soyabean oil yielded 96.6% biodiesel in 60 minutes using 3% CaO nanoparticles under 300W microwave [83]. In 15 minutes the yield was 56.2%.

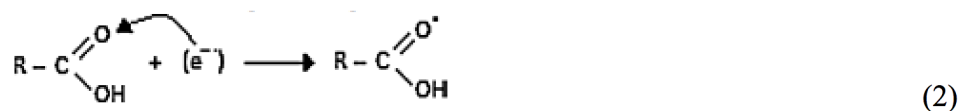
Dielectric constant of a solution is a critical parameter to be considered for biodiesel production using microwave irradiation. It was observed that using microwave, quantity of alcohol required can be reduced compared to the conventional process. It was also observed that a mixture of methanol and ethanol performed better under microwave irradiation [84]. Optimum reaction temperature should be in the range 50-60°C depending upon the boiling point of the alcohol used.

## 10. Enhanced photocatalytic synthesis of biodiesel

Solar energy is the ultimate source of energy on earth. Solar energy is the most abundantly available renewable energy while the world is facing a crisis of the



conventional energy. The term photocatalysis mean accelerating a chemical reaction assisted by light. When light having energy greater than the band gap of the semiconductor photocatalyst falls on it, an electron hole pair is generated which promotes the chemical reaction [85]. Photocatalysts enhance the reaction of conversion of free fatty acids to fatty acid alkyl esters [86]. In base-catalyzed transesterification reaction a proton is removed from the alcohol, the alcohol becomes more nucleophilic. Hence in a base-catalyzed reaction using NaOH as base, alcohol undergoes oxidation. If a heterogeneous semiconductor photocatalyst such as ZnO is used instead of base catalyst, the alcohol attracts the hole which is generated in by the semiconductor catalyst as sunlight or UV light falls on it. Due to high oxidative potential, the photo-generated holes ( $h^+$ ) may react with  $\text{CH}_3\text{OH}$  adsorbed on the catalyst surface, producing hydrogen ions ( $\text{H}^+$ ) and  $\text{CH}_2\text{O}^\bullet$  radicals. Thus the alcohol, a hole-scavenger, attracts the hole and gets oxidized. Photocatalytic oxidations of primary, secondary, tertiary alcohol have been studied with  $\text{TiO}_2$  particles suspended in aqueous solution. Thus the photocatalytically generated holes oxidize alcohol into carbonyl compound. Similar mechanism works in transesterification reaction. Alcohol (methanol) is oxidized by the  $\text{TiO}_2$  particles suspended in it under sunlight or UV light. Thus the  $\text{CH}_2\text{O}^\bullet$  radical reacts with the triglycerides, the free fatty acids present in vegetable oil to produce fatty acids of methyl esters. Photogenerated holes and electrons are consumed in the following way as proposed by Corro et al. [87].



Solar photocatalytic reactions have been popular and effective over time because the source of energy is completely inexpensive and environment friendly. But many of the photocatalytic reactions take place under UV lamp because light of ultraviolet spectrum may excite the photocatalyst with excitation energy higher than its band gap. Low to medium pressure mercury lamps within the UV range for photocatalysis degradation is used as well [88]. In different methods of waste glycerol conversion the test runs are performed in the batch and semi-batch reactors equipped with Xe lamps as a light source [89]. Much higher hydrogen productivity is observed for the Titania modified by noble metals than for transition metal used for modification. Thus UV light from Xenon lamps is another alternative for solar light. However sunlight still remains the most feasible source for conducting photo catalytic reactions. Corro et al. [87] reported the synthesis of biodiesel from *Jatropha curcus* seed oil using heterogeneous semiconductor photocatalyst such as ZnO,  $\text{SiO}_2$  and a mixture of  $\text{ZnO/SiO}_2$ . The photocatalysts were characterized by BET surface area, UV-vis spectrometry, FTIR and XRD analysis. Specific surface area of the  $\text{ZnO/SiO}_2$  was  $160 \text{ m}^2/\text{g}$ . The source of UV light was a 3W mercury lamp. They achieved about 100% conversion using a particular ratio of catalyst to oil keeping the methanol to oil ratio at 12:1. The activity of the catalyst remained unchanged even after 10 cycles.

Oloruntoba and Hamza [90] synthesized 45.2 % ZnO /49.6%  $\text{SiO}_2$  (by weight) by mechanochemical route and utilized it for photocatalytic transesterification of

waste cooking oil under visible light irradiation provided by a 500W halogen lamp; methanol: oil ratio was 12:1. A blank photochemical experiment was also conducted without the photocatalyst. Within 1 hour, yield was 87% for the photocatalytic process whereas for the photochemical process it was only 35%. The catalysts were characterized by XRD, UV-vis spectrometry, SEM and XRF.

TiO<sub>2</sub> has wide applications as a photocatalyst, but other than that when it is treated with H<sub>2</sub>SO<sub>4</sub> it forms SO<sub>4</sub><sup>-2</sup>/TiO<sub>2</sub> which also reportedly increases the yield of biodiesel through transesterification. Carlucci et al. [91], Chen et al. [92] and Peng et al. [93] reported this where TiO<sub>2</sub> can be used as a heterogeneous catalyst but not as a photocatalyst.

### 11. Combination of irradiation enhanced processes

There are a few reports on the simultaneous application of ultrasound and microwave irradiations for biodiesel production [94 – 96]. Martinez-Guerra and Gude [97] used sodium hydroxide catalyst at 0.75 wt%, methanol-to-oil ratio of 6:1 and the combined irradiation for 2 minutes to achieve a yield of 98%. Ardebili et al. [98] showed that the overall production cost decreased by using combined microwave and ultrasound. They have taken a catalyst concentration of 1.09% and a methanol-to-oil molar ratio of 7:3.1 at 58.4°C to get a yield of 97% in about 2 minutes.

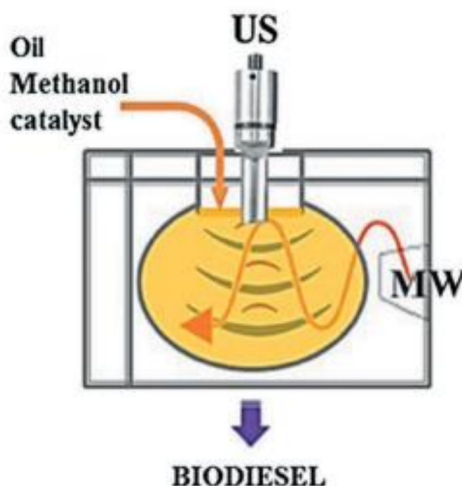


Fig. 8: Simultaneous use of microwave and ultrasonic for biodiesel production [97]

### 12. Comparison of the three irradiation-enhanced processes in biodiesel production

In terms of biodiesel production with different irradiation, main comparable factors are heating mechanism, reaction time, energy consumption etc. Transesterification reaction performed under conventional heating accompanied with mechanical mixing takes longer processing time (1-2 h) as heat energy is transferred to the raw materials through convection, conduction and radiation. High energy is consumed in conventional heating process. The reaction time is significantly reduced by applying ultrasound, microwave, photo-catalytic irradiation in transesterification of biodiesel.

Microwaves (2.45 GHz) produce dielectric heating at molecular level of raw materials which results the uniform heating and mixing in the reaction mixture at short time period. On the other hand, ultrasound induces continuous rarefaction and compression cycles of acoustic waves which produces micro bubbles to increase the activation energy as well as mass transfer rate of the reaction mixture, thus reduces the reaction time [99]. Ultraviolet lights can facilitate Transesterification reaction for synthesis of biodiesel. In such cases, a suitable photo-catalyst is required.  $\text{TiO}_2$ ,  $\text{ZnO}$  are among the efficient photo-catalysts used for production of biodiesel.

Refaat et al. [100] described the comparison of three processes in biodiesel production in his report. They produced 95.79% biodiesel in 60 minutes by conventional heating process at 6:1 methanol to waste vegetable oil molar ratio and 1% potassium hydroxide catalyst. This reaction was also carried out under ultrasound irradiation by keeping all reaction conditions same. 98.26% biodiesel was achieved in 5 min under this technique. Again, microwave irradiation was employed in the third set up with same reaction condition. 100% biodiesel yield was observed in 2 min. This was the clear evidence of irradiation mechanism as time saving, energy efficient and sustainable route of biodiesel production.

### 13. Conclusion and future perspectives

The present review summarized various irradiation-enhanced transesterification processes for production of biodiesel from vegetable oil and alcohol. Irradiations like light, ultrasound or microwave in presence of suitable catalysts can enhance the rate of reaction thereby reducing reaction time. Amount of catalysts and alcohols required are also reduced by irradiation improving the overall economy of the process. Combination of two irradiations may further enhance the rate and economy. However most of the available reports are of laboratory scale and scaling-up of irradiation-enhanced processes are challenges to the engineers rather than the scientists. Development of suitable catalysts suitable for each of the irradiation-assisted processes are also a major issue. Combination of conventional with irradiation-enhanced process or application of more than one irradiation may be examined for process intensification.

To combat air pollution due to the emissions generated out of the fossil fuels, especially petrol and diesel, mass production of biodiesel should be explored in an economic and environment-friendly way.

### References

- [1] J. Saez-Bastante, *Ultrasound-assisted biodiesel production* (Ph.D. thesis, University of Cordoba, Spain, 2015).
- [2] G. Knothe, *Inform* **11** (2001) 1103.
- [3] A.W. Schwab, M.O. Bagby and B. Freedman, *Fuel* **66** (1987) 1372.
- [4] A. Demirbas, A. Bafail, W. Ahmad and M. Sheikh, *Ener. Explor. Exploit.* **34** (2016) 290.
- [5] A.B.M.S. Hossain, A.N. Boyce, A. Salleh and S. Chandran, *Afri. J. Agri. Res.* **5** (2010) 1851.



- [6] A.K. Gupta., *J. Sci. Indus. Res.* **63** (2004) 39.
- [7] B.M. Agee, G. Mullins, D.J. Swartling, *Sustain. Chem. Proces.* **2** (2014) 1.
- [8] J. Saez-Bastante, S. Pinzi, I. Reyero, F. Priego-Capote, M.D. Luque de Castro and M.P. Dorado, *Fuel* **131** (2014) 6.
- [9] J.M. Avramovic, O.S. Stamenkovic, Z.B. Todorovic, M.L. Lazic and V.B. Veljkovic, *Fuel Proces. Tech.* **91** (2010) 1551.
- [10] A. Srivastava and R. Prasad, *Renew. Sustain. Ener. Rev.* **4** (2000) 111.
- [11] L. Canoira, J. Garcia Galean, R. Alcantara, M. Lapuerta and R. Garcia-Contreras, *Renew. Ener.* **35** (2010) 208.
- [12] B.K. De and D.K. Bhattacharyya, *Fett-Lipid.* **101** (1999) 404.
- [13] P.N. Giannelos, F. Zannikos, S. Stournas, E. Lois and G. Anastopoulos, *Indus. Crops Prod.* **16** (2002) 1.
- [14] P. Mazumdar, V.B. Borugadda, V.V. Goud, L. Sahoo, *Biom. Bioener.* **46** (2012) 546.
- [15] M. Canakci, J. van Gerpen, *Trans. Asae.* **44** (2001) 1429.
- [16] M.P. Dorado, E. Ballesteros, J.A. de Almeida, C. Schellert, H.P. Lohrlein and R. Krause. *Trans. Asae.* **45** (2002) 525.
- [17] A.B.M.S. Hossain and M.A. Mekhled, *Aust. J. Crop Sci.* **4** (2010) 543.
- [18] M. Mittelbach and C. Remschmidt, *Biodiesel: The comprehensive handbook* (BoersdruckGes. M.B.H., First edition, Biotechnology Journal, 102 2004).
- [19] I.M. Atadashi, M.K. Aroua, A.R.A. Aziz and N.M.N. Sulaiman, *Renew. Sustain. Ener. Rev.* **16** (2012) 3456.
- [20] I.B. Laskar, K. Rajkumari, R. Gupta, S. Chatterjee, B. Paul and L. Rokhum, *RSC Adv.* **8** (2018) 20131.
- [21] B. Freedman, E.H. Pryde and T.L. Mounts, *J. Am. Oil Chem. Soc.* **61** (1984) 1638.
- [22] G. Antolín, F.V. Tinaut, Y. Briceño, C. Pérez and A.I. Ramírez, *Bio. Tech.* **83** (2002) 111.
- [23] O. İlgen, A.N. Akin and N. Boz, *Turk. J. Chem.* **33** (2009) 289.
- [24] T. Iriany, O. Bani, S.Z.D.M. Parinduri and P.R.W. Ningsih, *Mater. Sci. Engg.* **309** (2018) 012107.
- [25] T.M. Mata and A.A Martins, *Current Tren. Chem. Engg.* (2010) 314.
- [26] F. Qiu, Y. Li, D. Yang, X. Li and P. Sun, *Appl. Ener.* **88** (2011) 2050.
- [27] M.B. Raphaela, A.O. Pâmella, C.M. Gabriel, V.A. André and P.B.C.F. Rodrigo, Ultrasound as a Metrological Tool for Monitoring Transesterification Kinetics, *Adv. Chem. Kine. Intech Open* (2018).
- [28] F. Trejo-Zárraga, F.D.J. Hernández-Loyo, J.C. Chavarria-Hernández and R. Sotelo-Boyás, Kinetics of transesterification processes for biodiesel production, In: *Biofuels: state of development*. Intech Open, pp 149–179 (2018).
- [29] H. Farouk, M.N.M. Jaafar and A.E. Atabani, *J. Teknol.* **69** (2014) 65.
- [30] M.Y. Koh and T.I.M. Ghazi, *Renew. Sustain. Ener.* **15** (2011) 2240.
- [31] D.Y.C. Leung, X. Wu and M.K.H. Leung, *Appl. Ener.* **87** (2010) 1083.

- [32] P.D. Patil and S. Deng, *Fuel* **88** (2009) 1302.
- [33] A. Singh, B. He, J. Thompson and J.V. Gerpen, *Am. Soc. Agri. Bio. Engg.* **22** (2006) 597.
- [34] U. Schuchardt, R. Sercheli and R.M. Vargas, *J. Braz. Chem. Soc.* **9** (1998) 199.
- [35] L.C. Meher, D.V. Sagar and S.N. Naik, *Renew. Sustain. Ener. Rev.* **10** (2006) 248.
- [36] F. Ahmed, S.O. Giwa, M. Ibrahim and A. Giwa, *Int. J. Chem. Tech. Res.* **9** (2016) 322.
- [37] C. Stavarache, M. Vinatoru, R. Nishimura and Y. Maeda, *Ultra. Sonochem.* **12** (2005) 367.
- [38] P.D. Patil, V.G. Gude, H.K. Reddy, T. Muppaneni and S. Deng, *J. Environ. Protect.* **3** (2012) 107.
- [39] S. Zheng, M. Kates, M.A. Dube and D.D. McLean, *Biomass Bioener.* **30** (2006) 267.
- [40] J.M. Marchetti, V.U. Miguel and A.F. Errazu, *Renew. Sustain. Ener. Rev.* **11** (2007) 1300.
- [41] D.A.G. Aranda, R.T.P. Santos, N. Tapanes and A.L.D. Ramos, *Catal. Lett.* **122** (2008) 20.
- [42] S.G. Bojan and S.K. Durairaj, *J. Sustain. Ener. Environ.* **3** (2012) 63.
- [43] E. Crabbe, C.N. Hipolito, G. Kobayashi, K. Sonomoto, K. and A. Ishizaki, *Process Biochem.* **37** (2001) 65.
- [44] S.V. Ghadge and H. Raheman, *Biomass Bioener.* **28** (2005) 601.
- [45] L.C. Meher, M.G. Kulkarni, A.K. Dalai and S.N. Naik, *Eur. J. Lipid Sci. Tech.* **108** (2006) 389.
- [46] J. Van Gerpen, B. Shanks, R. Pruzsko, D. Clements and G. Knothe, *Biodie. Produc. Tech.* (National Renewable Laboratory, 2004).
- [47] A.F. Lee, J.A. Bennett, J.C. Manayil and K. Wilson, *Chem. Soc. Rev.* **43** (2014) 7887.
- [48] P.K. Sahoo and L.M. Das, *Fuel* **88** (2009) 1588.
- [49] S. Siraj, B.M. Gitte, B.M. Joshi and H.M. Dharmadhikari, *Int. J. Eng. Res. Tech.* **2** (2013) 2077.
- [50] O.S. Tyagi, N. Atray, B. Kumar and A. Datta, *MAPAN-J. Meteor. Soc. Ind.* **25** (2010) 197.
- [51] K. Bansal, J. McCrady, A. Hansen and K. Bhalerao, *Fuel* **87** (2008) 3369.
- [52] E. Ali and C.I. Tay, *Procedia Eng.* **53** (2013) 7.
- [53] V.S. Moholkar, H.A. Choudhury, S. Singh, S. Khanna, A. Ranjan, S. Chakma and J. Bhasarkar, *Biofuels Biorefine.* **4** (2014) 35.
- [54] L. Buchori, I. Istadi and P. Purwanto, *Bull. Chem. React. Eng. Catal.* **11** (2016) 406.
- [55] M. Akram, A. Chowdhury and S. Chakrabarti, *Environ. Sci. Ind. J.* **12** (2016) 115.
- [56] K.G. Georgogianni, M.G. Kontominas, P.J. Pomonis, D. Avlonitis and V. Gergis, *Fuel Process. Tech.* **89** (2008) 503.
- [57] K.G. Georgogianni, A.K. Katsoulidis, P.J. Pomonis, G. Manos, M.G. Kontominas, *Fuel Process. Tech.* **90** (2009) 1016.
- [58] X. Fan, X. Wang and F. Chen, *J. Green Ener.* **7** (2010) 117.
- [59] F.F.P. Santos, L.J.B.L. Matos, S. Rodrigues and F.A.N. Fernandes, *Ener. Fuels* **23** (2009) 4116.

- [60] O. Babajide, L. Petrik, B. Amigun and F. Ameer, *Energies* **3** (2010) 1691.
- [61] A. Hayyan, M.A. Hashim, M.E.S. Mirghani, M. Hayyan and I.M. Alnashef, *J. Indus. Eng. Chem.* **20** (2014) 2066.
- [62] D. Kumar, G. Kumar, Poonam and C.P. Singh, *Ultra. Sonochem.* **17** (2010) 555.
- [63] X.U. Xuan and Y.C. Leung, *Linköping* (2011) 79.
- [64] V.G. Deshmane, P.R. Gogate and A.B. Pandit, *Indus. Eng. Chem.* **48** (2009) 7923.
- [65] L.T. Thanh, K. Okitsu, Y. Sadanaga, N. Takenaka, Y. Maeda and H. Bandow, *Biores. Tech.* **101** (2010) 639.
- [66] F.F.P. Santos, J.Q. Malveira, M.G.A. Cruz, M.G.A. and F.A.N. Fernandes, *Fuel* **89** (2010) 275.
- [67] H. Ding, W. Ye, Y. Wang, X. Wang, L. Li, D. Liu, J. Gui, C. Song and N. Ji, *Energy* **144** (2018) 957.
- [68] F. Motasemi and F.N. Ani, *Microwave irradiation biodiesel processing of waste cooking oil*, AIP Conference Proceedings, Malaysia, 842-853 (2012).
- [69] A. Arshanitsa, Y. Akishin, E. Zile, T. Dizhbite, V. Solodovnik and G. Telysheva, *Renew. Ener.* **91** (2016) 386.
- [70] Y.F. Huang, P.T. Chiueh and S.L. Lo, *Sustain. Environ. Res.* **26** (2016) 103.
- [71] S. Soltani, U. Rashid, R. Yunus and Y.H. Taufiq-yap, *Catal. Rev.* **57** (2015) 407.
- [72] Y. Xiang, Y. Xiang and L. Wang, *J. Taibah Univ. Sci.* **11** (2017) 1019.
- [73] V.G. Gude, P. Patil, E. Martinez-Guerra, S. Deng and N. Nirmalakhandan, *Sustain. Chem. Proces.* **1**, 5 (2013).
- [74] S.A.E. Sherbiny, A.A. Refaat, S.T.E. Sheltawy, *J. Adv. Res.* **1** (2010) 309.
- [75] V. Lertsathapornasuk, P. Ruangying, R. Pairintra and K. Krisnangkura, *Continuous transesterification of vegetable oils by microwave irradiation*, Proceedings of the 1st conference on energy network, Thailand (2005).
- [76] N. Azcan and O. Yilmaz, *Fuel* **104** (2013) 614.
- [77] G.M. Hincapié, S. Valange, J. Barrault, J.A. Moreno and D.P. López, *Univ. Sci.* **19** (2014) 193.
- [78] Y. Li, B. Ye, J. Shen, Z. Tian, L. Wang, L. Zhu, T. Ma, D. Yang and F. Qiu, *Biores. Tech.* **137** (2013) 220.
- [79] Y. Groisman and A. Gedanken, *J. Phys. Chem. C* **112** (2008) 8802.
- [80] J.J. Lin and Y.-W. Chen, *J. Taiwan Inst. Chem. Eng.* **75** (2017) 43.
- [81] H. Yuan, B. L. Yang and G. L. Zhu, *Ener. Fuels* **23** (2008) 548.
- [82] P. Khemthong, C. Luadthong, W. Nualpaeng, P. Changsuwan, P. Tongprem, N. Viriya-Empikul, and K. Faungnawakij, *Catal. Today* **190** (2012) 112.
- [83] M.C. Hsiao, C.-C. Lin and Y.-H. Chang, *Fuel* **90** (2011) 1963.
- [84] M.G. Kulkarni, A. K. Dalai and N. N. Bakhshi, *Biores. Tech.* **98** (2007) 2027.
- [85] D. Friedmann, A. Hakki, H. Kim, W. Choi and D. Bahnemann, *Green Chem.* **18** (2016) 5391.

- [86] Y. Asakuma, O. Kawanami, K. Maeda, H. Kuramochi and K. Fukui, *Int. J. Thermodyn.* **14** (2011) 193.
- [87] G. Corro, U. Pal and N. Tellez, *Appl. Catal. B: Environ.* **129** (2013) 39.
- [88] A. Sunthitikawinsakul and N. Sangatith, *Procedia Engg.* **32** (2012) 219.
- [89] M. Stelmachowski, M. Marchwickwa, E. Grabowska and M. Diak, *J. Adv. Oxid. Tech.* **17** (2016).
- [90] O. Oloruntoba and A. Hamza, *Food Environ. Safe.* **18** (2019) 89.
- [91] C. Carlucci, L. Degennaro and R. Luisi, *Catalysts* **9** (2019) 75.
- [92] C. Chen, L. Cai, X. Shangguan, L. Li, Y. Hong and G. Wu, *Roy. Soc. Open Sci.* **5** (2018)
- [93] B.X. Peng, Q. Shu, J.F. Wang, G.R. Wang, D.Z. Wang and M.H. Han, *Proces. Safe. Environ. Protec.* **86** (2008) 441.
- [94] M. Mostafaei, B. Ghobadian, M. Barzegar and A. Banakar, *Ultra. Sonochem.* **27** (2015) 54.
- [95] M. Sabzimaleki, B. Ghobadian, M.M. Farsibaf, G. Najafi, M.D. Soufi and S.M.S. Ardebili, *Chem. Prod. Proces. Model.* **10** (2015) 123.
- [96] P.R. Pandit and M.H. Fulekar, *J. Environ. Manage.* **198** (2017) 319.
- [97] E. Martinez-Guerra and V.G. Gude, *Fuel* **137** (2014) 100.
- [98] S. Ardebili, S. Mohammad, T.T. Hashjin, B. Ghobadian, G. Najafi, S. Mantegna and G. Cravotto, *Green Proces. Synth.* **4** (2015) 259.
- [99] H.D. Hanh, N.T. Dong, K. Okitsu, R. Nishimura and Y. Maeda, *Renew. Ener.* **34** (2009) 780.
- [100] A.A. Refaat and S.T.E Sheltawy, *WIT Transac. Ecol. Environ.* **109** (2008) 133.





## Factorization of the metric tensor in Kerr geometry

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**Abstract:** In Kerr spacetime, we use a null tetrad to factor the metric tensor in Boyer-Lindquist coordinates to construct a generator of the Lanczos potential for the conformal tensor.

**Keywords:** Kerr metric, Lanczos potential, Weyl tensor, Newman-Penrose formalism.

### 1. Introduction

Einstein's general relativity, which was proposed in 1915 for accounting gravitational effect on spacetime geometry, has been successful for demonstrating several phenomena related to astrophysics as well as cosmology. Basically, the vacuum solution of the Einstein field equations by Schwarzschild in the same year received an instant impact which however was based on the simplest assumption of static configuration and thus lacks of physical reality of rotation. Much later, in the year 1963, Kerr [1] introduced a rotation to the spherical system and thus provided a generalized solution to the Schwarzschild spacetime.

A good amount of subsequent works on the Kerr metric are available in literature [1 – 7] in connection to its different mathematical as well as physical structures and properties. On the other hand, an efficient mathematical tool is also available as background of the Kerr metric which is known as the Newman-Penrose formalism [8] which has been followed by several interesting works [9 – 18].

In the present investigation we use a null tetrad in the Kerr spacetime to factor the metric tensor in Boyer-Lindquist coordinates to construct a generator of the Lanczos potential for the conformal tensor. This has been carried out in Sec. 2 whereas we present some relevant comments on the result of our investigation in Sec. 3.

### 2. Mathematical Derivation

We shall employ the Newman-Penrose (NP) formalism [8 -- 18] with the null tetrad  $(l^\mu, n^\mu, m^\mu, \bar{m}^\mu)$  such that  $l^\alpha n_\alpha = -m^\alpha \bar{m}_\alpha = 1$ , then any metric tensor can be factored in the following form:

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$$g_{\alpha\beta} = l_{\alpha} n_{\beta} + l_{\beta} n_{\alpha} - m_{\alpha} \bar{m}_{\beta} - m_{\beta} \bar{m}_{\alpha} = 16 P_{\alpha}^{\mu} P_{\mu\beta}, \quad (1)$$

where

$$P_{\alpha}^{\mu} = \frac{1}{4} (m_{\alpha} m^{\mu} + \bar{m}_{\alpha} \bar{m}^{\mu} - l_{\alpha} l^{\mu} - n_{\alpha} n^{\mu}), \quad (2)$$

which can be applied to the Kerr metric [1 -- 7] in Boyer-Lindquist coordinates  $(t, r, \theta, \varphi)$  [10, 11, 13, 19 -- 22]:

$$ds^2 = \left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{\Sigma}{C} dr^2 - \Sigma d\theta^2 + \frac{4Mar}{\Sigma} \sin^2 \theta dt d\varphi - \left(r^2 + a^2 + \frac{2Ma^2r}{\Sigma} \sin^2 \theta\right) \sin^2 \theta d\varphi^2, \quad (3)$$

$$A = r - i a \cos \theta, \quad \Sigma = A\bar{A} = r^2 + a^2 \cos^2 \theta, \quad C = r^2 - 2Mr + a^2.$$

The interesting thing is that if we use the null tetrad:

$$\begin{aligned} (l^{\alpha}) &= \frac{1}{\sqrt{2\Sigma C}} (r^2 + a^2, -C, 0, a), & (n^{\alpha}) &= \frac{1}{\sqrt{2\Sigma C}} (r^2 + a^2, C, 0, a), \\ (m^{\alpha}) &= \frac{1}{\sqrt{2\Sigma}} \left(a, \sin \theta, 0, -i, \frac{1}{\sin \theta}\right), & (\bar{m}^{\alpha}) &= \frac{1}{\sqrt{2\Sigma}} \left(a, \sin \theta, 0, i, \frac{1}{\sin \theta}\right), \end{aligned} \quad (4)$$

then (2) acquires the structure:

$$(P_{\mu}^{\alpha}) = (P^{\alpha}_{\mu}) = \frac{1}{4} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (P_{\alpha\nu}) = (P_{\nu\alpha}) = \frac{1}{4} \begin{pmatrix} -g_{00} & 0 & 0 & -g_{03} \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & g_{22} & 0 \\ -g_{03} & 0 & 0 & g_{33} \end{pmatrix}, \quad (5)$$

and (5) turns out to be a generator of the Lanczos potential  $K_{\mu\nu\alpha}$  [23 -- 29] for the Weyl tensor  $C_{\mu\nu\alpha\beta}$ :

$$\begin{aligned} K_{\mu\nu\alpha} &= P_{\alpha\nu;\mu} - P_{\alpha\mu;\nu} + g_{\alpha\nu} A_{\mu} - g_{\alpha\mu} A_{\nu}, \quad (A_{\mu}) = \left(\frac{1}{3} P^{\nu}_{\mu;\nu}\right) = \left(0, \frac{r-M}{6C}, \frac{ctg \theta}{6}, 0\right), \\ K_{\mu\nu\alpha} &= -K_{\nu\mu\alpha}, \quad K_{\mu\nu\alpha} + K_{\nu\alpha\mu} + K_{\alpha\mu\nu} = 0, \end{aligned} \quad (6)$$

verifying the Lanczos gauges [23]:

$$K^{\mu\nu}_{;\nu} = 0, \quad K^{\mu\nu\alpha}_{;\alpha} = 0, \quad (7)$$

hence our potential (6) satisfies the Lanczos-Illge wave equation [30 -- 35] in this empty spacetime:

$$\square K_{\mu\nu\alpha} = 0. \quad (8)$$

The corresponding conformal tensor is generated via the expression:

$$\begin{aligned} C_{\mu\nu\alpha\beta} &= K_{\mu\nu\alpha;\beta} - K_{\mu\nu\beta;\alpha} + K_{\alpha\beta\mu;\nu} - K_{\alpha\beta\nu;\mu} + K_{\nu\alpha} g_{\mu\beta} - K_{\nu\beta} g_{\mu\alpha} + K_{\mu\beta} g_{\nu\alpha} - K_{\mu\alpha} g_{\nu\beta}, \\ K_{\mu\nu} &= K_{\nu\mu} = K_{\mu\alpha\nu}{}^{;\alpha}, \end{aligned} \quad (9)$$

which has type D in the Petrov classification [10, 11, 13, 27, 36].

The canonical null tetrad [37 -- 38] given in (4) implies the following spin coefficients:

$$\begin{aligned} \kappa = \sigma = \lambda = \nu = 0, \quad \rho = \mu = \frac{1}{A} \sqrt{\frac{C}{2\Sigma}}, \quad \pi = \tau = -\frac{a \sin \theta}{A \sqrt{2\Sigma}}, \quad \alpha = \beta = -\frac{a + i r \cos \theta}{2A \sqrt{2\Sigma} \sin \theta}, \\ \varepsilon = \gamma = \frac{a^2 - M r + i a (r-M) \cos \theta}{2A \sqrt{2\Sigma} C}, \quad \pi + \bar{\pi} = 2(\beta + \bar{\beta}), \quad \rho - \bar{\rho} = 2(\varepsilon - \bar{\varepsilon}), \quad \psi_2 = -\frac{M}{A^3} = 4(\varepsilon\rho - \pi\beta), \end{aligned} \quad (10)$$

thus (6) can be written in terms of the null tetrad:

$$\begin{aligned} K_{abc} + i {}^*K_{abc} = 2 [\Omega_0 (U_{ab} n_c + V_{ab} l_c) + \Omega_1 (M_{ab} (l_c + n_c) - U_{ab} m_c - V_{ab} \bar{m}_c) + \\ + \Omega_2 (V_{ab} n_c + U_{ab} l_c - M_{ab} (m_c + \bar{m}_c)) - \Omega_3 (V_{ab} m_c + U_{ab} \bar{m}_c)], \\ M_{ab} = m_a \times \bar{m}_b + n_a \times l_b, \quad U_{ab} = \bar{m}_a \times n_b, \quad V_{ab} = l_a \times m_b, \end{aligned} \quad (11)$$

with the NP components of the Lanczos potential:

$$\Omega_0 = \Omega_7 = -\frac{\pi}{4}, \quad \Omega_1 = \Omega_6 = -\left(\frac{\varepsilon}{3} + \frac{\rho}{12}\right), \quad \Omega_2 = \Omega_5 = -\left(\frac{\beta}{3} + \frac{\pi}{12}\right), \quad \Omega_3 = \Omega_4 = -\frac{\rho}{4}, \quad (12)$$

verifying (9), that is, the Weyl-Lanczos equations [17, 26 -- 29, 33, 39]:

$$\begin{aligned} \psi_0 &= 2[\delta\Omega_0 - D\Omega_4 + (-\bar{\alpha} - 3\beta + \bar{\pi})\Omega_0 + 3\sigma\Omega_1 + (\bar{\rho} + 3\varepsilon - \bar{\varepsilon})\Omega_4 - 3\kappa\Omega_5], \\ 2\psi_1 &= \Delta\Omega_0 + 3\delta\Omega_1 - \bar{\delta}\Omega_4 - 3D\Omega_5 - (3\gamma + \bar{\gamma} + 3\mu - \bar{\mu})\Omega_0 + 3(-\bar{\alpha} - \beta + \bar{\pi} + \tau)\Omega_1 + 6\sigma\Omega_2 + \\ &\quad + (3\alpha - \bar{\beta} + 3\pi + \bar{\tau})\Omega_4 + 3(\varepsilon - \bar{\varepsilon} - \rho + \bar{\rho})\Omega_5 - 6\kappa\Omega_6, \\ \psi_2 &= \Delta\Omega_1 + \delta\Omega_2 - \bar{\delta}\Omega_5 - D\Omega_6 - \nu\Omega_0 - (2\mu - \bar{\mu} + \gamma + \bar{\gamma})\Omega_1 + (-\bar{\alpha} + \beta + \bar{\pi} + 2\tau)\Omega_2 + \sigma\Omega_3 + \\ &\quad + \lambda\Omega_4 + (\alpha - \bar{\beta} + 2\pi + \bar{\tau})\Omega_5 - (\varepsilon + \bar{\varepsilon} - \bar{\rho} + 2\rho)\Omega_6 - \kappa\Omega_7, \\ 2\psi_3 &= 3\Delta\Omega_2 + \delta\Omega_3 - 3\bar{\delta}\Omega_6 - D\Omega_7 - 6\nu\Omega_1 + 3(\bar{\mu} - \mu - \bar{\gamma} + \gamma)\Omega_2 + (-\bar{\alpha} + 3\beta + 3\tau + \bar{\pi})\Omega_3 + \\ &\quad + 6\lambda\Omega_5 + 3(-\alpha - \bar{\beta} + \pi + \bar{\tau})\Omega_6 - (3\varepsilon + \bar{\varepsilon} - \bar{\rho} + 3\rho)\Omega_7, \\ \psi_4 &= 2[\Delta\Omega_3 - \bar{\delta}\Omega_7 - 3\nu\Omega_2 + (\bar{\mu} + 3\gamma - \bar{\gamma})\Omega_3 + 3\lambda\Omega_6 + (-3\alpha - \bar{\beta} + \bar{\tau})\Omega_7], \end{aligned} \quad (13)$$

and also the NP version of the Lanczos differential gauge (7):

$$\begin{aligned}
& \Delta\Omega_2 - \delta\Omega_3 - \bar{\delta}\Omega_6 + D\Omega_7 - 2\nu\Omega_1 + (3\mu + \bar{\mu} + \gamma - \bar{\gamma})\Omega_2 + (\bar{\alpha} - 3\beta + \tau - \bar{\pi})\Omega_3 + 2\lambda\Omega_5 + \\
& \quad + (-\alpha - \bar{\beta} + \bar{\tau} - 3\pi)\Omega_6 + (3\varepsilon + \bar{\varepsilon} - \rho - \bar{\rho})\Omega_7 = 0, \\
& \Delta\Omega_0 - \delta\Omega_1 - \bar{\delta}\Omega_4 + D\Omega_5 + (\mu + \bar{\mu} - 3\gamma - \bar{\gamma})\Omega_0 + (\bar{\alpha} + \beta - \bar{\pi} + 3\tau)\Omega_1 - 2\sigma\Omega_2 + \\
& \quad + (3\alpha - \bar{\beta} - \pi + \bar{\tau})\Omega_4 + (\bar{\varepsilon} - \varepsilon - \bar{\rho} - 3\rho)\Omega_5 + 2\kappa\Omega_6 = 0, \\
& -\Delta\Omega_1 + \delta\Omega_2 + \bar{\delta}\Omega_5 - D\Omega_6 + \nu\Omega_0 + (\gamma + \bar{\gamma} - 2\mu - \bar{\mu})\Omega_1 + (-\bar{\alpha} + \beta + \bar{\pi} - 2\tau)\Omega_2 + \sigma\Omega_3 - \\
& \quad - \lambda\Omega_4 + (-\alpha + \bar{\beta} + 2\pi - \bar{\tau})\Omega_5 + (-\varepsilon - \bar{\varepsilon} + 2\rho + \bar{\rho})\Omega_6 - \kappa\Omega_7 = 0.
\end{aligned} \tag{14}$$

The factorization (1) gave the generator (2) of the Lanczos spin tensor, which allowed obtaining the vector (6):

$$(A_\mu) = \left(\frac{1}{3} P^\nu_{\mu;\nu}\right) = \left(0, \frac{r-M}{6C}, \frac{ctg\theta}{6}, 0\right) = (f_\mu), \quad f = \frac{1}{12} \ln(C \sin^2\theta), \quad \square f = 0, \tag{15}$$

so it is interesting to see that the following function:

$$F = \ln \psi_2^{1/6} + 3f, \tag{16}$$

generates four spin coefficients:

$$\varepsilon = DF, \quad \gamma = -\Delta F, \quad \beta = \delta F, \quad \alpha = -\bar{\delta} F, \tag{17}$$

in accordance with Nerozzi-Elbracht [40].

We note the Bianchi identities [10, 11, 13, 17, 27, 39, 41]:

$$\rho = DG, \quad \mu = -\Delta G, \quad \tau = \delta G, \quad \pi = -\bar{\delta} G, \quad G = \ln \psi_2^{1/3}, \tag{18}$$

and the relations:

$$Df = -\Delta f = \frac{M-r}{6\sqrt{2}\Sigma C} = \frac{1}{6}(2\varepsilon - \rho), \quad \delta f = -\bar{\delta} f = -\frac{ictg\theta}{6\sqrt{2}\Sigma} = \frac{1}{6}(2\beta - \pi). \tag{19}$$

We can use (5), (6) and the Christoffel symbols for the Kerr metric [42] [or to apply (4), (10) and (12) in (11)] to obtain the non-zero components of the Lanczos potential:

$$\begin{aligned}
K_{abc} &= \frac{1}{2} (g_{0c} \Gamma^0_{ab} + g_{3c} \Gamma^3_{ab}) - g_{ac} A_b, \quad a, c = 0, 3, \quad b = 1, 2, \\
K_{121} &= \frac{\Sigma}{6C} ctg\theta, \quad K_{122} = \frac{\Sigma(M-r)}{6C},
\end{aligned} \tag{20}$$



that is:

$$K_{b00} = -\frac{1}{4}g_{00,b} + g_{00}A_b, \quad K_{b03} = K_{b30} = -\frac{1}{4}g_{03,b} + g_{03}A_b, \quad K_{b33} = -\frac{1}{4}g_{33,b} + g_{33}A_b, \quad b = 1, 2, \quad (21)$$

$$K_{211} = g_{11} A_2, \quad K_{122} = g_{22} A_1,$$

where we observe that  $K_{03\mu} = 0, \quad \forall \mu$ .

### 3. Conclusion

To summarize, in the present work we have shown derivation of the Lanczos potential through factorization of the metric tensor in Kerr geometry on purely analytical basis. The possible physical meaning of the Lanczos potential in general relativity, is an open problem. We believe that  $K_{\mu\nu\alpha}$  behaves in the asymptotic region as a density for the angular momentum of a rotating black hole. It would be therefore justified if one could find out a few applications of this result. Though at this moment we have no specific applications of our result, however, we would like to mention that in a future project it is possible to study the importance of Lanczos potential in astrophysical as well as cosmological models with angular momentum, i.e., with rotation as in Kerr's black hole.

### References

- [1] R.P. Kerr, *Phys. Rev. Lett.* **11** (1963) 237.
- [2] E. Newman and A. I. Janis, *J. Math. Phys.* **6** (1965) 915.
- [3] B. Léauté, *Ann. Inst. Henri Poincaré A* **8** (1968) 93.
- [4] M.M. Schiffer, R.J. Alder, J. Mark and Ch. Sheffield, *J. Math. Phys.* **14** (1973) 52.
- [5] S.K. Bose, *J. Math. Phys.* **16** (1975) 772.
- [6] J. Enderlein, *Am. J. Phys.* **65** (1997) 897.
- [7] S.A. Teukolsky, *Class. Quantum Gravit.* **32** (2015) 124006.
- [8] E. Newman and R. Penrose, *J. Math. Phys.* **3** (1962) 566.
- [9] S.J. Campbell and J. Wainwright, *Gen. Relativ. Gravit.* **8** (1977) 987.
- [10] M. Carmeli, *Group theory and general relativity* (McGraw-Hill, New York, 1977).
- [11] S. Chandrasekhar, *The mathematical theory of black holes* (Clarendon Press, Oxford, 1983).
- [12] S. R. Czapor, R. G. McLenaghan, *Gen. Relativ. Gravit.* **19** (1987) 623.

- [13] H. Stephani, D. Kramer, M. MacCallum, C. Hoenselaers and E. Herlt, *Exact solutions of Einstein's field Equations* (Cambridge University Press, 2003).
- [14] E. Newman and R. Penrose, *Scholarpedia* **4** (2009) 7445.
- [15] A.H. Hasmani, *J. Sci.* **1** (2010) 82.
- [16] A.H. Hasmani and P.I. Andharia, *J. Dynam. Syst. Geom. Theor.* **9** (2011) 27.
- [17] P. Lam, J. López-Bonilla, R. López-Vázquez and A.K. Rathie, *Prespacetime J.* **6** (2015) 684.
- [18] J. López-Bonilla, R. López-Vázquez, J. Morales and G. Ovando, *Prespacetime J.* **6** (2015) 697.
- [19] R.H. Boyer and R.W. Lindquist, *J. Math. Phys.* **8** (1967) 265.
- [20] M. Visser, *arXiv: 0706.0622 [gr-qc]*.
- [21] T. Mueller and F. Grave, *arXiv: 0904.41.84 [gr-qc]*.
- [22] T. Adamo and E. Newman, *arXiv: 1410.6626 [gr-qc]*.
- [23] C. Lanczos, *Rev. Mod. Phys.* **34** (1962) 379.
- [24] C. Lanczos, *Ann. Facul. Sci. Univ. Clermont-Ferrand* **8** (1962) 167.
- [25] Z. Perjés, *The works of Kornél Lánzos on the theory of relativity*, in “A panorama of Hungarian mathematics in the twentieth century. I”, Ed. J. Horváth (Springer-Verlag, Berlin, 2006).
- [26] G. Ares de Parga, O. Chavoya and J. López-Bonilla, *J. Math. Phys.* **30** (1989) 1294.
- [27] P. O'Donnell, *Introduction to 2-spinors in general relativity* (World Scientific, Singapore, 2003).
- [28] P. O'Donnell and H. Pye, *EJTP* **7** (2010) 327.
- [29] Z. Ahsan, *The potential of fields in Einstein's theory of gravitation* (Springer, Singapore, 2019).
- [30] R. Illge, *Gen. Relativ. Gravit Proc. Roy. Soc. London A* **447** (1994) 557.
- [32] S. B. Edgar, *Mod. Phys. Lett. A* **9** (1994) 479.
- [33] S.B. Edgar and A. Höglund, *Proc. Roy. Soc. London A* **453** (1997) 835.
- [34] J. López-Bonilla, J. Morales and G. Ovando, *Prespacetime J.* **6** (2015) 269.

- [35] S. Álvarez-Ballesteros, R. Cruz-Santiago and J. López-Bonilla, *Prespacetime J.* **8** (2017) 18.
- [36] V. Barrera-Figueroa, J. López-Bonilla, R. López-Vázquez and S. Vidal-Beltrán, *Prespacetime J.* **7** (2016) 445.
- [37] J. López-Bonilla, R. López-Vázquez, J. Morales and G. Ovando, *Prespacetime J.* **7** (2016) 1176.
- [38] J. López-Bonilla, R. López-Vázquez, H. Torres-Silva, *Prespacetime J.* **7** (2016) 1194.
- [39] P. Lam, J. López-Bonilla, R. López-Vázquez and S. Vidal-Beltrán, *World Sci. News* **96** (2018) 1.
- [40] A. Nerozzi and O. Elbracht, *arXiv: 0811.1600 [gr-qc]*.
- [41] D.E. Rowe, *Math. Intelligen.* **24** (2002) 57.
- [42] V.P. Frolov and I.D. Novikov, *Black Hole Physics: Basic concepts and new developments* (Springer, Berlin, 1998).



## Dynamical and geometrical analysis of scale factor based solution in general relativity

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**Abstract:** In this paper, we have derived Einstein's field equations in an isotropic spacetime. The dynamical parameters are derived with assumed scale factors and analyzed. All the models are tested through geometrical diagnostic methods and luminosity distance. The stability analysis on the assumptions made are also performed. A detail derivation of all the dynamical parameters and other relevant parameters are done.

**Keywords:** General Relativity; Hubble Parameter; Dynamical Parameter; Geometrical Parameter

### 1. Introduction

In modern cosmology, the main theme of research is on the issue of late time cosmic acceleration phenomena. Supernovae type Ia provides strong evidence that the universe is currently undergoing an accelerated phase of expansion. Plethora of observations during the last two decades have confirmed the late time cosmic acceleration of the universe [1, 2, 3, 4, 5, 6, 7]. These observations have developed a curiosity among the cosmologists to explain this late time dynamics. In the purview of General Relativity (GR), it becomes difficult to explain this issue. Therefore, the idea of an unusual and exotic dark energy (DE) form with negative pressure has been surfaced. The most intriguing thing is that, so far we do not exactly know the nature and origin of this exotic energy source. The contribution of DE as compared to the baryonic matter and dark matter in providing an anti gravity effect to drive apart the universe for acceleration is the maximum. Recent Planck data estimates a lion share of 68.3 percent in favour of DE [8, 9, 10]. The late time cosmic dynamics and the consequent dark energy is understood through a dark energy equation of state parameter  $\omega = \frac{p}{\rho}$ , where  $p$  and  $\rho$  respectively denote the pressure and energy density of dark energy. There are approaches to study the dark energy models, (i) geometrically modifying the gravity or (ii) incorporating dark energy pressure and energy density in the matter part. We prefer the second approach here.

In this project, we have derived the field equations of Einstein's field equation in FRW space-time and the energy momentum tensor is that of dark energy. In the field equations we have incorporated scale factors to get the background cosmology. In Sec. 2, we have derived the field equations. In Sec. 3, the solutions of the field equations using scale factors are obtained. The stability analysis and the luminosity distance are obtained for the models in Secs. 4 and 5 respectively. Concluding remarks are given in Sec. 6.

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## 2. Spacetime and field equations

In order to construct the cosmological model of the universe, we have considered flat Friedmann-Robertson-Walker spacetime (FRW) in the form

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2 + dz^2), \quad (1)$$

where  $a = a(t)$  be the average scale factor.

The spherical symmetric form of Eq. (1) can be written as

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2). \quad (2)$$

The Einstein's field equations can be expressed as

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = \kappa T_{ij}, \quad (3)$$

where  $G_{ij}$ ,  $R_{ij}$ ,  $R$ ,  $g_{ij}$ ,  $T_{ij}$  respectively represent the Einstein tensor, Ricci tensor, Ricci scalar, invariant and energy momentum tensor. The invariant  $g_{ij}$  can be represented in matrix form as

$$g_{ij} = \begin{pmatrix} a^2 & 0 & 0 & 0 \\ 0 & a^2 r^2 & 0 & 0 \\ 0 & 0 & a^2 r^2 \sin^2 \theta & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (4)$$

The contravariant form of  $g^{ij}$  can be calculated as

$$g^{ij} = \frac{\text{cofactor of } g_{ij} \text{ in } g}{|g_{ij}|} \quad (5)$$

$$g^{ij} = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 & 0 \\ 0 & \frac{1}{a^2 r^2} & 0 & 0 \\ 0 & 0 & \frac{1}{a^2 r^2 \sin^2 \theta} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (6)$$

From the matrix, we can calculate the Christoffel symbols as

$$\Gamma_{bc}^a = \Gamma_{bc,k}^a = \frac{g^{ak}}{2} \left[ \frac{\partial g_{bk}}{\partial x^c} + \frac{\partial g_{ck}}{\partial x^b} - \frac{\partial g_{bc}}{\partial x^k} \right]. \quad (7)$$

The surviving Christoffel symbols are:

$$\begin{aligned} \Gamma_{14}^1 &= \frac{\dot{a}}{a}, & \Gamma_{22}^1 &= -r, & \Gamma_{33}^1 &= -r \sin^2 \theta, & \Gamma_{12}^2 &= \frac{1}{r}, \\ \Gamma_{24}^2 &= \frac{\dot{a}}{a}, & \Gamma_{33}^2 &= -\sin \theta \cos \theta, & \Gamma_{13}^3 &= \frac{1}{r}, & \Gamma_{23}^3 &= \cot \theta, \\ \Gamma_{34}^3 &= \frac{\dot{a}}{a}, & \Gamma_{11}^4 &= a\dot{a}, & \Gamma_{22}^4 &= a\dot{a}r^2, & \Gamma_{33}^4 &= a\dot{a}r^2 \sin^2 \theta. \end{aligned}$$

Now, the Ricci tensor  $R_{ij}$  can be calculated using the formula

$$R_{ij} = R_{ij,a}^a = \frac{\partial}{\partial x^j} \Gamma_{ib}^b - \frac{\partial}{\partial x^b} \Gamma_{ij}^b + \Gamma_{ib}^a \Gamma_{ja}^b - \Gamma_{ij}^a \Gamma_{ab}^b. \quad (8)$$

For the spacetime 2, the Ricci tensor can be calculated as

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$$\begin{aligned}
R_{11} &= -a^2 \left( \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} \right), \\
R_{22} &= -a^2 r^2 \left( \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} \right), \\
R_{33} &= -a^2 r^2 \sin^2 \theta \left( \frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} \right), \\
R_{44} &= 3\frac{\ddot{a}}{a}.
\end{aligned} \tag{9}$$

Now, the Ricci Scalar  $R = g^{ij} R_{ij}$  can be calculated as

$$R = -6 \left( \frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} \right). \tag{10}$$

The energy momentum tensor can be described in the form of perfect fluid as

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}, \tag{11}$$

where  $p$  and  $\rho$  respectively denote the pressure and energy density of the matter field. Now, the non-vanishing energy momentum tensor can be calculated as

$$\begin{aligned}
T_{11} &= p a^2, \\
T_{22} &= p a^2 r^2, \\
T_{33} &= p a^2 r^2 \sin^2 \theta, \\
T_{44} &= \rho.
\end{aligned}$$

Now, we can obtain the Einstein's field equations (3) in the following form

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\kappa p, \tag{12}$$

$$3\frac{\dot{a}^2}{a^2} = \kappa \rho. \tag{13}$$

### 3. Solution of the field Equations

In order to understand the background cosmology, we need to incorporate the scale factor to obtain the solution to the field equations. We have studied the cosmological model by incorporating two types of scale factor such as (i) quasi linear scale factor, (ii) linear combinations of scale factor and (iii) bouncing scale factor.

#### 3.1 Quasi linear scale factor

The quasi linear scale factor can be expressed in the form  $a = e^{H_0 t} - \frac{M^2 t^2}{12}$ , where  $H_0$  is the present value of the Hubble parameter and  $M$  be a constant. Subsequently the Hubble parameter  $H$  can be derived as  $H = \frac{\dot{a}}{a} = H_0 - \frac{M^2 t}{6}$ .

Incorporating this form of quasi linear scale factor in Einstein's field equations (12) – (13), we can derive the pressure, energy density as

$$p = \frac{1}{\kappa} \left[ \frac{\frac{1}{3}H_0 M^2 t e^{H_0 t} + \frac{M^2}{3} e^{H_0 t} + \frac{M^2}{6} H_0^2 t^2 e^{H_0 t} - \frac{M^4 t^2}{18} - 3H_0^2 e^{2H_0 t}}{(e^{H_0 t} - \frac{M^2 t^2}{12})^2} \right], \quad (14)$$

$$\rho = \frac{1}{\kappa} [3H^2] = \frac{3}{\kappa} \left[ \frac{H_0 e^{H_0 t} - \frac{M^2 t}{6}}{e^{H_0 t} - \frac{M^2 t^2}{12}} \right]^2 = \frac{3}{\kappa} \left[ \frac{H_0^2 e^{2H_0 t} + \frac{M^4 t^2}{36} - \frac{H_0}{3} M^2 t e^{H_0 t}}{e^{2H_0 t} + \frac{M^4 t^4}{144} - \frac{M^2}{6} t^2 e^{H_0 t}} \right] \quad (15)$$

Subsequently the equation of state (EoS) parameter  $\omega = \frac{p}{\rho}$  can be calculated as

$$\omega = \frac{p}{\rho} = -\frac{2}{3} \left[ \frac{H_0^2 e^{2H_0 t} - \frac{1}{12} H_0^2 e^{H_0 t} M^2 t^2 - \frac{1}{6} M^2 e^{H_0 t} + \frac{1}{72} M^4 t^2}{(H_0 e^{H_0 t} - \frac{M^2 t}{6})^2} + \frac{1}{3} \right]. \quad (16)$$

The Null Energy Condition (NEC), Strong Energy Condition (SEC) and Dominant Energy Condition (DEC) can be respectively calculated as,

$$\rho + p = \frac{M^4 t^2 + 6M^2 e^{H_0 t} ((H_0 t - 2)^2 - 2)}{36\kappa \left( e^{H_0 t} - \frac{M^2 t^2}{12} \right)^2}, \quad (17)$$

$$\rho + 3p = \frac{6}{\kappa} \left[ \frac{\frac{M^2}{6} - H_0^2 e^{H_0 t}}{e^{H_0 t} - \frac{M^2 t^2}{12}} \right], \quad (18)$$

$$\rho - p = \frac{2}{\kappa} \left[ \frac{3H_0^2 e^{2H_0 t} - \frac{H_0^2 e^{H_0 t} M^2 t^2}{12} - \frac{2}{3} H_0 e^{H_0 t} M^2 t - \frac{1}{6} M^2 e^{H_0 t} + \frac{5}{72} M^4 t^2}{(e^{H_0 t} - \frac{M^2 t^2}{12})^2} \right] \quad (19)$$

The deceleration parameter, scalar expansion and the spatial volume can be obtained as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{-(H_0^2 e^{2H_0 t} - \frac{e^{H_0 t} M^2}{6} - \frac{M^2 H_0^2 e^{H_0 t} t^2}{12} + \frac{M^4 t^2}{72})}{H_0^2 e^{2H_0 t} - \frac{2M^2 t H_0 e^{H_0 t}}{6} + \frac{M^4 t^2}{36}}, \quad (20)$$

$$\theta = 3H = 3\frac{\dot{a}}{a} = 3\frac{\left[ H_0 e^{H_0 t} - \frac{M^2 t}{6} \right]}{\left[ e^{H_0 t} - \frac{M^2 t^2}{12} \right]}, \quad (21)$$

$$V = a^3 = \left[ e^{H_0 t} - \frac{M^2 t^2}{12} \right]^3. \quad (22)$$

The geometrical diagnostic, state finder pair ( $r$ ,  $s$ ) can be obtained as

$$r = \frac{\ddot{H}}{H^3} - (2 + 3q) = \frac{a^2 \ddot{a}}{\dot{a}^3} \quad (23)$$

$$\begin{aligned} &= \frac{H_0^3 e^{3H_0 t} + \frac{M^4 H_0^3 t^4 e^{H_0 t}}{144} - \frac{M^2 H_0^3 t^2 e^{2H_0 t}}{6}}{H_0^3 e^{3H_0 t} + \frac{M^4 H_0 t^2 e^{H_0 t}}{12} - \frac{3M^2 t H_0^2 e^{2H_0 t}}{6} - \frac{M^6 t^3}{216}}, \\ s &= \frac{r-1}{3(q-0.5)} = \frac{2[\ddot{a}a^2 - \dot{a}^2]}{3\dot{a}[-2a\ddot{a} - \dot{a}^2]} \quad (24) \\ &= \frac{2\left[\left(\frac{tH_0^2}{2} - \frac{t^2 H_0}{6}\right)M^2 e^{H_0 t} + \left(H_0^3 t^4 - \frac{5H_0 t^2}{12}\right)M^2 e^{H_0 t} + \frac{M^6 t^3}{216}\right]}{\left[H_0^3 e^{3H_0 t} - \left(\frac{H_0^3 t^2}{6} - \frac{H_0^2 t}{6} + \frac{H_0}{3}\right)M^2 e^{2H_0 t} + \left(\frac{H_0^3 t^2}{36} - \frac{H_0^2 t}{18} + \frac{t}{18}\right)M^4 e^{2H_0 t}\right]}. \end{aligned}$$

Another geometric diagnostic methods is the  $Om(z)$  diagnostic that involves first derivative of the scale factor and therefore becomes easier to apply to distinguish between different dark energy models. The  $Om(z)$  parameter can be defined as

$$Om(z) = \frac{E^2(z) - 1}{(1+z)^3 - 1} = \frac{\dot{z}^2 - H_0(1+z)^2}{H_0^2(1+z)^2 \left[ (1+z)^3 - 1 \right]} \quad (25)$$

$$= \frac{\left( e^{H_0 t - \frac{M^2 t^2}{12}} \right) \left( H_0 e^{H_0 t - \frac{M^2 t}{6}} \right)^2 - H_0^2 \left( e^{H_0 t - \frac{M^2 t^2}{12}} \right)}{H_0^2 \left( 1 - e^{H_0 t - \frac{M^2 t^2}{12}} \right)},$$

where  $E(z) = \frac{H(z)}{H_0}$  is the dimensionless Hubble parameter and  $H_0$  is the Hubble rate at present epoch. Also, the red shift can be expresses with respect to the scale factor as,  $z = \frac{1}{a} - 1$ . Now, the throat radius of Morris-Thorne wormhole can be obtained as

$$R = -\frac{\kappa a}{2c\dot{a}} = -\frac{\kappa(e^{H_0 t - \frac{M^2 t^2}{12}})}{2c(H_0 e^{H_0 t - \frac{M^2 t}{6}})}. \quad (26)$$

It has been observed that when  $t \rightarrow 0$ , then  $R \rightarrow -\frac{k}{2cH_0}$  whereas when  $t \rightarrow \infty$ , then  $R \rightarrow -\frac{k}{2cH_0}$ .  $R = 0$  when  $H_0 t - 2\ln t = \ln \frac{M^2}{12}$ .

### 3.2 Linear combination of exponential scale factor

Another important scale factor to study the background cosmology is the linear combination of exponential scale factor which can be described as  $a(t) = \sigma e^{\lambda t} + \tau e^{-\lambda t}$ . Unlike in exponential scale factor, this scale factor consisting of both positive and negative exponential factor. The significance of the scale factor is that when  $\sigma = 0$ , the scale factor reduces to negative exponential factor whereas for  $\tau = 0$ , it reduces to positive exponential function. The Hubble parameter  $H$  can be obtained as  $H = \frac{\dot{a}}{a} = \frac{\sigma \lambda e^{\lambda t} - \tau \lambda e^{-\lambda t}}{\sigma e^{\lambda t} + \tau e^{-\lambda t}}$  and subsequently  $\dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = \frac{4\sigma\tau\lambda^2}{(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2}$ . Now from the Einstein's field equations in FRW space-time, the pressure, energy density and EoS parameter  $\omega = \frac{p}{\rho}$  can be obtained as

$$p = -\frac{\lambda^2}{\kappa} \left[ \frac{3\sigma^2 e^{2\lambda t} + 3\tau^2 e^{-2\lambda t} + 2\sigma\tau}{(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2} \right], \quad (27)$$

$$\rho = \frac{3\lambda^2}{\kappa} \left[ \frac{\sigma e^{\lambda t} - \tau e^{-\lambda t}}{\sigma e^{\lambda t} + \tau e^{-\lambda t}} \right]^2, \quad (28)$$

$$\omega = -\frac{1}{3} \left[ \frac{3\sigma^2 e^{2\lambda t} + 3\tau^2 e^{-2\lambda t} + 2\sigma\tau}{(\sigma e^{\lambda t} - \tau e^{-\lambda t})^2} \right]. \quad (29)$$



The deceleration parameter, scalar expansion and the volume of the model can be obtained as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2}{(\sigma e^{\lambda t} - \tau e^{-\lambda t})^2}, \quad (30)$$

$$\theta = 3H = 3\frac{\dot{a}}{a} = 3\left(\frac{\sigma\lambda e^{\lambda t} - \tau\lambda e^{-\lambda t}}{\sigma e^{\lambda t} + \tau e^{-\lambda t}}\right), \quad (31)$$

$$V = a^3 = \left(\sigma e^{\lambda t} + \tau e^{-\lambda t}\right)^3. \quad (32)$$

The Null Energy Condition (NEC), Strong Energy Condition (SEC) and Dominant Energy Condition (DEC) for the linear combinations of exponential scale factor can be respectively calculated as

$$\rho + p = -\frac{2}{\kappa}\dot{H} = \frac{-8\sigma\tau\lambda^2}{\kappa(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2}, \quad (33)$$

$$\rho + 3p = \frac{6}{\kappa}(\dot{H} + H^2) = -\frac{6}{\kappa}\lambda^2, \quad (34)$$

$$\rho - p = \frac{2}{\kappa}[\dot{H} + 3H^2] = \frac{2\lambda^2}{\kappa} \left[ \frac{3\sigma^2 e^{2\lambda t} + 3\tau^2 e^{-2\lambda t} - 2\sigma\tau}{(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2} \right]. \quad (35)$$

The state finder diagnostic pair  $(r, s)$  can be calculated as

$$r = \frac{\ddot{H}}{H^3} - (2 + 3q) = \frac{a^2\ddot{a}}{a^3\dot{a}^3} = \left(\frac{\sigma e^{\lambda t} + \tau e^{-\lambda t}}{\sigma e^{\lambda t}\tau e^{-\lambda t}}\right)^2, \quad (36)$$

$$s = \frac{r - 1}{3(q - 0.5)} = \frac{2[\ddot{a}a^2 - \dot{a}^2]}{3\dot{a}[-2a\ddot{a} - \dot{a}^2]} = \frac{-8\sigma\tau}{3(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2}. \quad (37)$$

It can be noted here that if the diagnostic pair  $(r, s)$  approaches to  $(1, 0)$  at late time of the cosmic evolution, the model is said to be  $\Lambda_{CDM}$  model. Now the other geometrical diagnostic  $Om(z)$  diagnostic can be calculated as

$$Om(z) = \frac{E^2(z) - 1}{(1+z)^3 - 1} = \frac{\dot{z}^2 - H_0(1+z)^2}{H_0^2(1+z)^2[(1+z)^3 - 1]}, \quad (38)$$

$$\text{where } E(z) = -\frac{\dot{z}}{H_0(1+z)} = \frac{\dot{a}}{H_0 a} = \frac{\sigma\lambda e^{\lambda t} - \tau\lambda e^{-\lambda t}}{H_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})}.$$

The throat radius for this case can be obtained a

$$R(t) = -\frac{k(\sigma e^{\lambda t} + \tau e^{-\lambda t})}{2c\lambda(\sigma e^{\lambda t} - \tau e^{-\lambda t})}. \quad (39)$$

It can be noted that when  $t \rightarrow 0$ , then  $R \rightarrow -\frac{k(\sigma+\tau)}{2c\lambda(\sigma-\tau)}$  whereas when  $t \rightarrow \infty$ , then  $R \rightarrow -\frac{k}{2c\lambda}$ . Also  $R$  vanishes for  $2\lambda t = \ln\left(-\frac{\tau}{\sigma}\right)$ .

### 3.3 Bouncing scale factor

Before the onset of inflation where the universe undergoes an almost exponential expansion, singularity occurs and therefore inflationary scenario fails to reconstruct the complete past history of the universe. As a solution to the challenges faced by the inflationary scenario, [11,12] proposed the matter bounce scenario. In the matter bounce cosmology, the initial singularity can be suitably avoided. In this scenario, matter dominates the universe at the bounce epoch and therefore it is possible to generate the density fluctuation comparable to that of observations [13, 14]. In bouncing cosmologies there is an initial phase of contraction where matter dominates the universe followed by a bounce without any singularity and then there is a causal generation for fluctuation. In this section, we will investigate the bouncing model in the presented framework to explore the geometrical degrees of freedom to explain the late time cosmic speed up phenomenon. This will also enable us to investigate the bouncing behavior at an initial epoch. In order to investigate the dynamics under the purview of bouncing scenario, we have considered the dynamics of the models through the presumed bouncing scale factor, in the form  $a(t) = a_0 e^{(t-t_0)^{2n}}$ , where  $a_0 > 0$ ,  $n \neq 0$ . Subsequently the hubble parameter can be obtained as  $H = 2n(t-t_0)^{2n-1}$ . As in the previous cases, in this case also, we can derive the matter pressure  $p$ , energy density  $\rho$  and EoS parameter  $\omega = \frac{p}{\rho}$  from (12) – (13) as

$$p = -\frac{1}{\kappa} [2\dot{H} + 3H^2] = -\frac{4n}{\kappa} (t-t_0)^{2n-2} [(2n-1) + 3n(t-t_0)^{2n}], \quad (40)$$

$$\rho = \frac{3}{\kappa} (4n^2 (t-t_0)^{4n-2}), \quad (41)$$

$$\omega = -\frac{(t-t_0)^{-2n}}{3n} [(2n-1) + 3n(t-t_0)^{2n}]. \quad (42)$$

The NEC, SEC and DEC for the bouncing scale factor can be obtained as

$$\rho + p = -\frac{1}{\kappa} 4n(2n-1)(t-t_0)^{2n-2}, \quad (43)$$

$$\rho + 3p = -\frac{6}{\kappa} [4n^2 (t-t_0)^{4n-2} + 2n(2n-1)(t-t_0)^{2n-2}], \quad (44)$$

$$\rho - p = \frac{4}{\kappa} [n(2n-1)(t-t_0)^{2n-2} + 6n^2 (t-t_0)^{4n-2}]. \quad (45)$$

The physical parameters such as the deceleration Parameter, scalar expansion and the volume can be calculated for the proposed bouncing scale factor as

$$q = -\left[1 + \frac{(2n-1)(t-t_0)^{-2n}}{2n}\right], \quad (46)$$

$$\theta = 6n(t-t_0)^{2n-1}, \quad (47)$$

$$V = a_0^3 e^{3(t-t_0)^{2n}}. \quad (48)$$

The parameters of geometrical diagnostics such as state finder pair  $(r, s)$  and the  $om(z)$  can be derived as

$$r = 1 + \frac{3(2n-1)(t-t_0)^{-2n}}{2n} + \frac{(2n-1)(2n-2)(t-t_0)^{-4n}}{4n^2}, \quad (49)$$

$$s = \frac{-(2n-1)(t-t_0)^{-2n} [6n + (2n-2)(t-t_0)^{-2n}]}{6n[(2n-1)(t-t_0)^{-2n} - 3n]} \quad (50)$$

and

$$Om(z) = \frac{E^2(z) - 1}{(1+z)^3 - 1} = \frac{\dot{z}^2 - H_0(1+z)^2}{H_0^2(1+z)^2 [(1+z)^3 - 1]}, \quad (51)$$

where  $E(z) = -\frac{\dot{z}}{H_0(1+z)} = \frac{\dot{a}}{H_0 a} = \frac{2n(t-t_0)^{2n-1}}{H_0}$ .

The throat radius for the bouncing scale factor can be calculated as

$$R = -\frac{\kappa a}{2c\dot{a}} = -\frac{\kappa}{4nc(t-t_0)^{2n-1}}. \quad (52)$$

It can be observed that, when  $t \rightarrow 0$ ,  $R = -\frac{\kappa}{4nc(-t_0)^{2n-1}}$  and when  $t \rightarrow \infty$ ,  $R \rightarrow 0$ .

#### 4. Stability analysis

In order to claim that our constructed cosmological models are stable, we will undertake the stability analysis of the cosmological models presented in the previous section. This can be obtained by analyzing  $C_s^2 = \frac{dp}{d\rho}$ . It is worthy to mention here that if  $C_s^2 > 0$ , the model is stable and for  $C_s^2 < 0$ , the model is unstable. Hence we can write

$$\frac{dp}{d\rho} = \frac{12\sigma\tau}{(\sigma e^{\lambda t} + \tau e^{-\lambda t})^2 ((\sigma e^{\lambda t} - \tau e^{-\lambda t})^2 - 1)(1 - (\sigma e^{\lambda t} + \tau e^{-\lambda t})^2)}, \quad (53)$$

$$\frac{dp}{d\rho} = \frac{3n(t-t_0)^{2n}}{(n-1) + 3n(t-t_0)^{2n}}. \quad (54)$$

#### 5. Luminosity distance

In this section, we have discussed the cosmological parameters by calculating the Luminosity distance. This can be calculated as

$$d_L(z) = r_1(1+z)a_0 = a_0(1+z) \int \frac{dt}{a}. \quad (55)$$

The luminosity distance for quasi linear scale factor, linear combination of exponential scale factor and bouncing scale factor can be respectively calculated as

$$d_L(z) = a_0(1+z) \int \frac{dt}{e^{H_0 t} - \frac{M^2 t^2}{12}}, \quad (56)$$

$$d_L(z) = a_0(1+z) \int \frac{dt}{\sigma e^{\lambda t} + \tau e^{-\lambda t}}, \quad (57)$$

$$d_L(z) = a_0(1+z) \int \frac{dt}{a_0 e^{(t-t_0)^{2n}}}. \quad (58)$$

The distance modulus can be calculated as

$$\mu(z) = 5 \log d_L(z) + 25. \quad (59)$$

The distance modulus for quasi linear scale factor, linear combination of exponential scale factor and bouncing scale factor can be respectively calculated as

$$\mu(z) = 5 \log \left[ a_0(1+z) \int \frac{dt}{e^{H_0 t} - \frac{M^2 t^2}{12}} \right] + 25, \quad (60)$$

$$\mu(z) = 5 \log \left[ a_0(1+z) \int \frac{dt}{\sigma e^{\lambda t} + \tau e^{-\lambda t}} \right] + 25, \quad (61)$$

$$\mu(z) = 5 \log \left[ a_0(1+z) \int \frac{dt}{a_0 e^{(t-t_0)^{2n}}} \right] + 25. \quad (62)$$

## 6. Conclusion

The physical parameters of the cosmological models are derived using the scale factors (i) quasi linear (ii) linear combination of exponential function, (iii) bouncing scale factors. The equation of state parameter, from where the nature of universe during evolution would be known, has been derived with respect to the cosmic time. The energy conditions for these three models are derived along with the physical parameters such as deceleration parameter, scalar expansion, volume, state finder pair are also derived with respect to the cosmic time. The stability analysis are performed along with the determination of luminosity distance. As a future work to this project, the graphical representation of the parameters can be presented and its behavior can be analyzed.

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## References

1. S. Perlmutter et al., Nature **391** (1998) 51.
2. A.G. Reiss et al., Astron. J. **116** (1998) 1009.
3. R. Knop et al., Astrophys. J. **598** 102 (2003).
4. D.J. Eisenstein et al., Astrophys. J. **633** 560 (2005).
5. D.N. Spergel et al., Astrophys. J. S. **170** 377 (2007).
6. M. Sullivan et al., Astrophys. J. **737** 102 (2011).
7. N. Suzuki et al., Astrophys. J. **746** 85 (2012).
8. P.A.R. Ade et al., ( Planck Collaboration) arXiv:1303.5076 [astro-ph.CO].
9. P.A.R. Ade et al., ( Planck Collaboration) arXiv:1303.5082 [astro-ph.CO].
10. P.A.R. Ade et al., ( Planck Collaboration) arXiv:1303.5084 [astro-ph.CO].
11. R. Brandenberger, Int. J. Mod. Phys. Conf. Ser. **01** (2011) 67.
12. R. Brandenberger, arXiv:1206.4196 (2012).
13. M. Novello and S.E. Perez Bergliaffa, Phys. Rep. **463** (2008) 127.
14. R. Brandenberger and P. Peter, Found. Phys. **47** (2017) 797.



## **C.T.R. Wilson and the Nobel Prize in Physics**

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**Abstract:** C.T.R. Wilson (1869 – 1959), in 1927 shared the Physics Nobel Prize with A.H. Compton. Documents regarding C.T.R. Wilson's nomination for the Nobel Prize were obtained from the archive of the Nobel Foundation. Results of their analysis are given in the present communication.

**Keywords:** C.T.R. Wilson; D.M. Bose; A.H. Compton, Nobel Prize in Physics

### **1. Introduction**

The Scottish physicist Charles Thomson Rees Wilson (1869–1959), constructed an apparatus to produce clouds under laboratory conditions. In 1911, Wilson took the first photograph of ionized particles, and proved that the cloud chamber was suitable for the investigations of radioactivity and atomic structure [1]. There are a number of articles on Wilson's life and science [2]. However, none of them deals with his nomination for the prestigious Nobel Prize. The present article fulfils the gap.

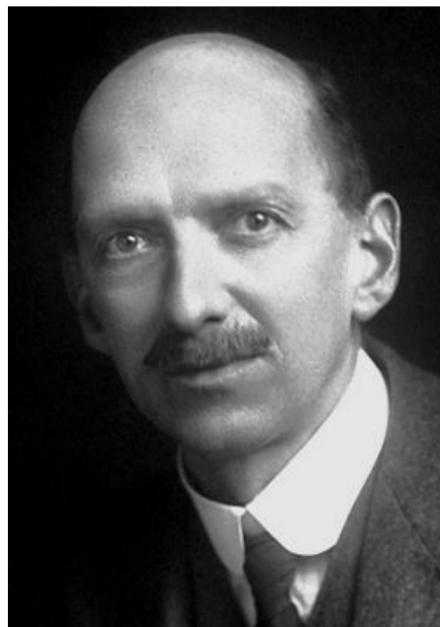


Fig. 1: C.T.R. Wilson [*Credit: Wiki – Public domain*]

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To start with a short review of the work of the Indian physicist D.M. Bose, who improved Wilson's cloud chamber to study radioactive radiation, is given. It shows that between 1914 and 1919, he made similar observations, for which C.T.R. Wilson was awarded the Nobel Prize.

## 2. D.M. Bose and cloud chamber research work in Europe and Kolkata

D.M. Bose's life and work are discussed by S.C. Roy and R. Singh in different articles and books [3]. The following summary is taken from them.

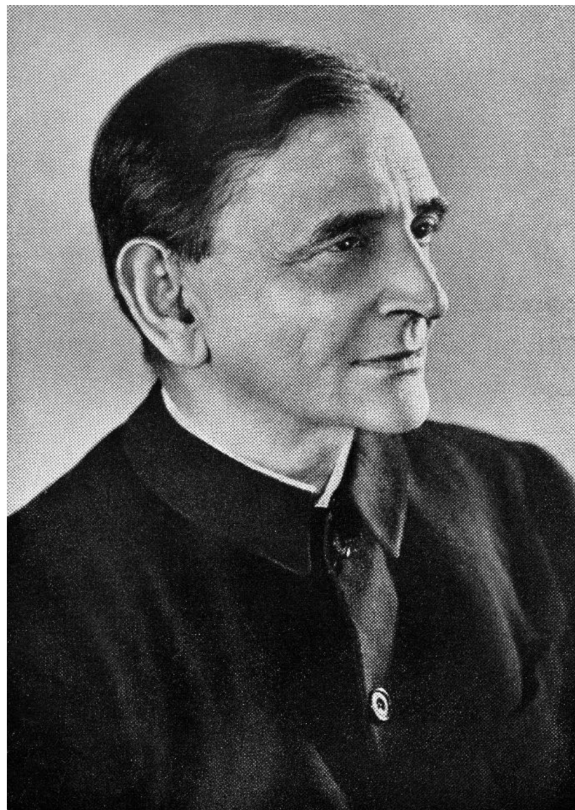


Fig. 2: D.M. Bose [Credit: Bose Institute]

In 1907, D.M. Bose joined Christ's College, Cambridge. In the laboratory of J. J. Thomson, he studied the nature of the condensed nuclei produced by incandescent platinum wire in glass bulb saturated with water vapours. Bose obtained A.R.C.S. Diploma and B.Sc. (first class) in Physics, from the Royal College of Science, London. After staying for a short time in Calcutta, young D.M. Bose as a research scholar went to Berlin for higher studied. In Berlin, his research guide was, E. Regener, who in the "Naturwissenschaften" emphasized the importance of Wilson's cloud chamber [4].

In Berlin, Bose improved Wilson's cloud chamber. He replaced some of the brass parts by glass. For instance, the piston was substituted by a glass valve of large cross-section. With the improved expansion chamber, Bose showed: (i) the existence of H-particles, which come into play after the collision of  $\alpha$ -particles with hydrogen atoms,

(ii) made the path of the colliding and deflecting  $\alpha$ - and  $\beta$  particles visible and (iii) observed the ionisation traces of gamma radiation in hydrogen [5].

In 1919 Bose returned back to India. He was the first person, who constructed Wilson cloud chamber at the University of Calcutta. With S.K. Ghosh he wrote: “On the photographing the ionization tracks of the rest atoms of radioactive elements.” They photographed the tracks of the “rest atoms”, and confirmed the shape of ionisation tracks of rest atoms as predicted by L. Wertenstein [6]. They also studied tracks of  $\alpha$ -particles in helium. “Besides the long range recoil helium atoms, we have obtained a few photographs in which are shown the ionisation tracks of all the constituent parts of a helium atom, namely, of the nucleus and the two bound electrons”, wrote the authors [7].

D.M. Bose in a letter of May 10, 1923, wrote to the Nobel Laureate E. Rutherford, that his explanation for the curved tracks of slow  $\beta$ -particles is more or less as suggested by A.H. Compton. Further, he told Rutherford that in Berlin:

“I had obtained  $\alpha$ -particle tracks of similar type also. I thought that such discontinuities of tracks were due to the rapid alternation of charge of the ionizing particles – a phenomenon whose possibility had been pointed out by (L.) Flamm. But Prof. Regener was of opinion that they were rather due to the fluctuation in the distribution of water vapours along the tracks.” (D.M. Bose to E. Rutherford, May 10, 1923).

In his letter, Bose sent some photographs of observations, and wrote: “I have also obtained one case in which a single electron is ejected from the point where a helium nucleus has started its track.”

Shortly after the discoveries by A.H. Compton and C.T.R. Wilson (detail below), E. Regener published the detailed summary of Bose’s thesis “Study of the passage of  $\alpha$ - and  $\beta$ -particles through gases” [8].

The Austrian Marietta Blau acclaimed Bose to be the first scientist who observed H-particles with cloud chamber. This brought her to the idea to visualize cosmic rays with emulsion [9]. K.W.F. Kohlrausch gave Bose credit to be the first person to study beta particles with cloud chamber [10]. T. Alper acknowledged to H.A. Bumstead, C.T.R. Wilson and Bose for making the  $\gamma$  radiation visible with cloud chamber [11].

### 3. C.T.R. Wilson –Scientific work and nomination for the Nobel Prize

Main part of Wilson’s work on the behaviour of ions as condensation nuclei was carried out between 1895 and 1900. As is evident from his list of publications, between 1901 and 1908, he mainly concentrated on atmospheric electricity and condensation phenomena [12]. In 1910, he constructed a new type of expansion chamber in order to observe the tracks of individual  $\alpha$ - and  $\beta$ -particles. The results produced with it were not satisfactory [13]. In 1923, the cloud chamber was improved. He made investigations on X-rays and  $\beta$ -particles. He was able to observe individual quanta of radiation, i.e., under suitable conditions identified electrons ejected by an atom. With that he proved A.H. Compton’s view that “a single electron may be effective in scattering a quantum of radiation and that in so doing it receives the whole momentum of the quantum” [14]. In

the case of nitrogen, Wilson quantitatively confirmed Rutherford's theory of nuclear deflection, which predicted a definite amount of charge of deflecting nucleus [15].

On Jan. 7, 1915, R.J. Strutt (Lord Rayleigh) wrote a letter to the Nobel Committee and nominated Wilson "for his researches on the condensation of water vapours on gaseous ions (which have led to the methods in use for measuring the ionise charge,) and on the track of the ionising rays from radioactive bodies." In his second letter of Jan. 5, 1915, Lord Rayleigh submitted copies of Wilson's important papers. NC did not take notice of Strutt's nomination. The Physics Nobel Prize was awarded to W.H. Bragg and his son W.L. Bragg, who, in 1912, discovered that the structure of crystals could be determined by X-rays. In 1918, Edward B. Poulton, Oxford, nominated William Crookes and C.T.R. Wilson for a shared prize [16]. Between 1919 and 1923, Wilson was not proposed.

In 1923, A.H. Compton, U.S.A., showed that when X-rays are scattered by electrons, due to inelastic collisions, there is an increase in the wavelength of the rays. During the process, a part of the energy is transferred to the recoiling electrons. With the experiment, he proved the quantum nature of radiation [17]. C.T.R. Wilson with the improved cloud chamber reconfirmed Compton's results. He also proved that his chamber was "able to deal with individual quanta of radiation, in the sense that the track of the electron ejected from the atom which emits the quantum of radiation and that of the electron ejected from the atom which absorbs the radiation may under suitable conditions be identified" [18].

For the Physics Nobel Prize for the year 1924, Wilson was nominated by his countrymen Hugh F. Newall, E. Rutherford, and J.J. Thomson. On Dec. 17, 1923, H.F. Newall wrote to the Nobel Committee that already in 1911, C.T.R. Wilson discovered a method to make the ionised particles visible. In the following years he improved the apparatus to observe the atomic disintegration. Newall stated:

"At a time when the indirect methods had perhaps yielded their harvest of results, this impressive procedure came to provide immediate confirmation and extension, and to push direct scrutiny of the history of the individual flying electron or ion into new developments. The photographic records of their life histories and interactions have ever since been indispensable to research in this domain." (H.F. Newall to the Nobel Committee, Dec. 17, 1923).

After telling the importance of Wilson's work in the field of meteorology and cosmic rays, he stated that Wilson's

"recent papers condense the results of many years work on the effects of X-rays and on the activities exhibited along the tracks of  $\alpha$ -rays and  $\beta$ -rays, both directly and in relation to the study of the characteristic radiations of elementary substances under the action of X-rays; they will be a valuable mine for future investigators in atomic science." (H.F. Newall to the Nobel Committee, Dec. 17, 1923).

E. Rutherford and J.J. Thomson in a joint letter emphasized Wilson's work in the field of meteorology. They wrote that Wilson's tracking method:

"...is based on his own discovery, made many years ago, that ions produced in gases act as condensation nuclei under certain conditions. Photographs of these trails show in a vivid and concrete way the processes occurring in the passage of



ionizing radiations through gases.” (J.J. Thomson, E. Rutherford to the Nobel Committee, Dec. 4, 1923).

They were also of the opinion that Wilson’s photographic method had not only confirmed the general correctness of the existing ionisation and scattering theories, but also hinted that we are still far from understanding the complicated processes that occur due to the interaction of radiation like X-rays and a gas.

For the year 1924, the Nobel Prize was reserved as none of the nominees was seen as candidate, who deserves the Prize.

For the Nobel Prize for the year 1925, the Director of Radium Institute in Vienna, Stefan Meyer proposed Swede Manne Siegbahn and C.T.R. Wilson for the Nobel Prize (S. Meyer to the Nobel Committee, Dec. 29, 1924). Other persons who nominated M. Siegbahn were M. von Laue and E.P. Jordan [19].

The reserved prize from the year 1924 was awarded, in 1925, to the Manne Siegbahn “for his discoveries and research in the field of X-ray spectroscopy” [20]. The prize for the year 1925 was awarded jointly to James Franck and Gustav L. Hertz “for their discovery of the laws governing the impact of an electron upon an atom” [21].

Before we proceed further with the case of Wilson, it will be worthwhile to say a few words about A.H. Compton’s nomination, who shared the Nobel Prize with Wilson.

#### **4. A.H. Compton’s nomination**

In 1923, A.H. Compton showed that when X- rays are scattered by electrons; due to inelastic collisions, there is an increase in the wavelength of the rays. During the process, a part of the energy is transferred to the recoiling electrons [22].

The first time, A.H. Compton was nominated, for the prize for the year 1925, by Henry G. Gale from Chicago. A year later, R.A. Millikan, A. Einstein, W. Wien, C. Fabry, and W.L. Bragg proposed Compton [23]. In 1927, E.P. Jordan, M. von Laue, H.F. Osborn, M. Planck, C.D. Walcott and W. Wien proposed Compton for the unshared prize. M. Born and J. Franck were of the opinion that either Compton alone should get the award, or it should be divided between W. Gerlach and O. Stern. R. Pohl proposed that Compton and C. Ramsauer should share the prize [24].

Wilson’s nominators were the following Nobel Laureates: Jean Perrin from France, J.J. Thomson, and E. Rutherford from U.K [25]. The French physicist J. Perrin in a letter of Jan. 26, 1926, to the Nobel Committee gave credit to Wilson for establishing a technique, with which the trajectories of the individual alpha- and beta particles can be made visible. He stated that Wilson’s method can be used to understand the mechanics of ionisation of gases by X-rays. This mechanism was studied in detail in the laboratory of P. Auger, France. In U.K., Blackett’s work in Rutherford’s laboratory proved that the same method allows us, to study the genesis of atoms from its individual parts, under the influence of shooting alpha particles. (J. Perrin to the Nobel Committee, Jan. 26, 1926).

J.J. Thomson’s letter, written on Jan. 26, 1927, was rather short. He emphasised that Wilson’s method is the most direct and in some cases the only method to

investigate the properties of the X-rays,  $\alpha$ - and  $\beta$  particles. It had helped much for the progress of modern physics.

E. Rutherford was determined to put Wilson's case more strongly. On Jan. 27, 1927, he wrote three and half pages long "Statement of claims of Professor C.T.R. Wilson FRS." Therein he stated:

"I think all physicists would agree that his pioneer work on the conditions of formation of nuclei in dust free moist air is of the highest originality and merit. The subsequent development of this discovery to show the tracks of individual alpha and beta particles and the effects produced by the passage of X-rays and other ionizing radiations through a gas, was a masterly piece of work carried out with the technical perfection and refinement so characteristic of Professor Wilson's researches."

In order to emphasize the affect of Wilson's method on the contemporary research Rutherford observed in the "Statement of claims ...":

"The experiments of Wilson himself and later of A.H. Compton, (W.) Bothe, (J.M.) Nuttall and (E.J.) Williams and others have shown that the scattering of quantum is accompanied by the motion of the electron involved, and that this is in the direction and magnitude to be expected on the theory. It would have been difficult to obtain such information by any other method - data which are essential for the verification of the theory."

In "Statement of claim ...", Rutherford called Wilson's cloud chamber method as the "Experimentum crucis" for the modern Physics. After naming Wilson's method as the most powerful tool to attack and solve the fundamental problems, he referred to his nominator's work in the field of meteorology and added, "All of this work is characterized not only by great originality and experimental skill, but the results are interpreted with judgement and vision."

The Nobel Committee asked its expert to prepare a supplementary report on Wilson's scientific work (details below).

### **5. Evaluation of Wilson's work by the expert M. Siegbahn**

The Nobel Committee asked Manne Siegbahn to evaluate Wilson's achievements. In order to keep the length of article appropriate, only some of the highlights of the Report of Expert, dated Aug. 9, 1927, (original is Swedish) are given below.

"As stated in the previous report (i.e., year 1924) Wilson's method makes it possible to study in details the phenomena that are related to the collisions of  $\alpha$ -particles in gas molecules. Also the method has been used to visualise a number of properties of radiation related to its quantum nature."

"Using cloud chamber, (P.) Auger studied the secondary betaparticles, which come into existence due to the interaction of gas atoms and X-ray. Auger has shown that Wilson's method is an excellent substitute to the other techniques, and in some particular cases it can be used to get more direct information, than had been possible by other methods."



“Wilson’s method is complimentary to Scintillation and Geiger’s methods used for the study of nucleus and  $\alpha$ -particle collision, and the path of particle traversed by it. The observations regarding collision of nucleus and  $\alpha$ -particles, in which the photon is ejected from a nuclei (a rare phenomenon as observed by Rutherford) was made by Paul M.S. Blackett, and W.D. Harkins and (H.A.) Shadduck (...) using Wilson’s method.”

“In the end it must be stated that Wilson’s method is also applicable for the study of Compton effect. Although the effect is assured by spectroscopic measurements, yet the measurement of velocity and direction of electrons with the help of Wilson’s method are of great value to explain this phenomenon.”

The opinion of the Nobel Committee about Wilson’s achievements is given in the following section.

### **6. Opinion of the Nobel Committee**

The Report to the Nobel Committee, dated Sept. 30, 1927 (henceforth RNC, original in Swedish) shows that for the year 1927, 20 nominators proposed 34 nominees. The Committee reported that Wilson, who since 1924 was nominated, this time had been supported by the three Nobel Laureates. The Nobel Committee opined:

“From this supplemented inquiry it is clear through the investigations made by Auger at the laboratory of Perrin, that Wilson’s method is a most valuable complement to other methods. In certain cases his method gives more pertinent information than is possible by other methods. The method has been applied in several research projects, among other the study of the Compton effect. Although this effect was verified through spectroscopy after the first demonstration through ionisation methods, the demonstrations by Wilson have been of great value for the clarification of the phenomenon.”

The general overall view of the Committee was:

“On the whole the method has shown to be a most valuable contribution to the experimental methods of radioactivity and related subjects. The full significance has been revealed only in recent time, and the committee members consider the importance so great as to warrant the Nobel Prize in physics.”

The NC proposed to the Swedish Academy of Sciences that: “the 1927 price in physics should be shared in equal parts between Prof. A.H. Compton, Chicago, for his discovery of the effect named after him, and Prof. C.T.R. Wilson, Cambridge, for his method of making the paths of electrically charged particles visible by condensation of vapour.” The Swedish Academy of Sciences, which is the final authority to decide on the Prize, also gave its adjudication as proposed by the Committee.

### **7. Conclusions**

D.M. Bose learned to construct and work with the cloud chamber in the U.K. and Germany. Based on the knowledge, he constructed the instruments at the University of Calcutta. Bose’s case shows the transfer of technology and methodology from one country to other.

European scientists, such as, K.W.F. Kohlrausch, T. Alper, M. Blau, and E. Regener appreciated Bose's work. They even gave credit to Bose being the first in observing some effect. However, none of the Europeans nominated Bose for the Nobel Prize. Bose had no chance to win the Nobel Prize, because "no one can receive the prize in a given year without being nominated for that year" [26]. However, the fact remains that D.M. Bose along with S.K. Mitra were the first two Indian physicists, who were asked by the Nobel Committee to send proposals. In a joint letter, they nominated M.N. Saha. Details of Saha's nomination and the opinion of the Nobel Committee are explored elsewhere [27].

C.T.R. Wilson was often nominated for the Nobel Prize, but he was ignored. In 1927 NC or rather the Swedish Academy gave decision in his favour, because the full significance of Wilson's cloud chamber was revealed only after 1926.

The number of A.H. Compton's nominators was much higher than C.T.R. Wilson, still the NC proposed for a shared Nobel Prize. This suggests that the number of nominations is irrelevant as far as the decision of the NC is concerned.

### Acknowledgements

I thank the Archive of the Nobel Foundation, Stockholm for the reports of Nobel Committee, expert, and the nomination letters referred to in this paper. Thanks are due to the Churchill Archive Cambridge for E. Rutherford's "Statement of claims of Professor C.T.R. Wilson FRS." I acknowledge with pleasure that the Archive Humboldt-University Berlin provided me with D.M. Bose's documents referred to in the article. With pleasure I acknowledge the help given by Mrs. Adelheid W. Demmer, ICBM – University of Oldenburg, Professor Olaf Beckman, Department of Materials Science, University of Uppsala for Swedish translation. Thanks are due to Mrs. Rosemarie Engels for French translation of J. Perrin's letter. Thanks are due to Prof. Michael Komorek, Head of Research Group: Physics Didactic and Science Popularisation, University of Oldenburg, for providing research facilities. Last but not the least, I thank Dr. S. Ray, Editor, Scientific Voyage for editorial work.

### References

- [1] C.T.R. Wilson, *Proc. R. Soc. Lond.* **85** (1911) 285.
- [2] G.L.E. Turner, C.T.R. Wilson, in: *Dictionary of scientific biography, vol. 14*, Gillispie C.C. (Editor-in-Chief) (Charles Scribner's Sons, New York 1976, pp. 420-423); C.F. Powell, C.T.R. Wilson, in: *Selected papers of Cecil Frank Powell*, Burhop E.H.S. (Ed.) (North-Holland Publ. Comp., Amsterdam 1972, pp.357-368); P.M.S. Blackett, *Biogr. Mem. Fell. R. Soc. Lond.* **6** (1960); C.T.R. Wilson, *Notes and Records R. Soc. Lond.* **14** (1960) 163.
- [3] R. Singh, *D.M. Bose - His scientific work in international context* (Shaker Verlag, Aachen 2016); S.C. Roy and R. Singh, *D.M. Bose - A scientist incognito* (Bose Institute Publication, Kolkata 2017); S.C. Roy and R. Singh, *Sci. Cult.* **82** (2016) 364; R. Singh and S.C. Roy, *Sci. Cult.* **83** (2017) 364.
- [4] E. Regener, *Naturwiss.* **1** (1913) 299.
- [5] D. Bose, *Phys. Z.* **17** (1916) 388.
- [6] D.M. Bose and S.K. Ghosh, *Phil. Mag.* **45** (1923) 1050.
- [7] D.M. Bose and S.K. Ghosh, *Nature* **111** (1923) 463.

- [8] D. Bose, *Zeit. Phys.* **12** (1923) 207.
- [9] M. Blau, *Sitzun. Akad. Wissen. Wien, IIa* **134** (1925) 427.
- [10] K.W.F. Kohlrausch, *Radioaktivität* (Akad. Verlag. M.B. H., Leipzig 1928, p. 28).
- [11] T. Alper, *Zeit. Phys.* **76** (1932) 172.
- [12] P.I. Dee and T.W. Wormell, *Notes and Records R. Soc. Lond.* **18** (1963) 54.
- [13] C.T.R. Wilson, *Proc. R. Soc. Lond. A* **85** (1911) 285.
- [14] C.T.R. Wilson, *Proc. R. Soc. Lond. A* **104** (1923) 1.
- [15] C.T.R. Wilson, *Proc. R. Soc. Lond. A* **104** (1923) 192.
- [16] G. Küppers, P. Weingart and N. Ulitzka., *Die Nobelpreise in Physik und Chemie. 1901-1929, Materialien zum Nominierungsprozeß, Report Wissenschaftsforschung Nr. 23* (B. Kleine Verlag, Bielefeld 1982, p. 38).
- [17] A.H. Compton, *Phys. Rev.* **21** (1923) 483.
- [18] C.T.R. Wilson, *Proc. R. Soc. Lond. A* **104** (1923) 1.
- [19] R.M. Friedman, *The politics of excellence – Behind the Nobel Prize science* (A.W.H. Freeman Book, New York 2001, p. 154).
- [20] <https://www.nobelprize.org/prizes/physics/1924/summary/>
- [21] <https://www.nobelprize.org/prizes/physics/1925/summary/>
- [22] A.H. Compton, *Phys. Rev.* **21** (1923) 483.
- [23] G. Küppers, P. Weingart and N. Ulitzka., *Die Nobelpreise in Physik und Chemie. 1901-1929, Materialien zum Nominierungsprozeß, Report Wissenschaftsforschung Nr. 23* (B. Kleine Verlag, Bielefeld 1982, pp. 46 – 47).
- [24] E. Crawford, J.L. Heilbron and R. Ullrich, *The Nobel Population 1901-1937 – A Census of the Nominators and Nominees for the Prizes in Physics and Chemistry* (University of California, Berkeley 1987, pp. 106-107).
- [25] E. Crawford, J.L. Heilbron and R. Ullrich, *The Nobel Population 1901-1937 – A Census of the Nominators and Nominees for the Prizes in Physics and Chemistry* (University of California, Berkeley 1987, p. 108).
- [26] H. Nielsen and K. Nielsen, Summary and conclusions, in: (Eds. Nielsen H., Nielsen K., *Neighbouring Nobel – The history of thirteen Danish Nobel Prizes* (Aarhus University Press, Aarhus 2001, pp. 553-587).
- [27] R. Singh, *Asian J. Phys.* **27** (2018) 647.



## OBITUARY

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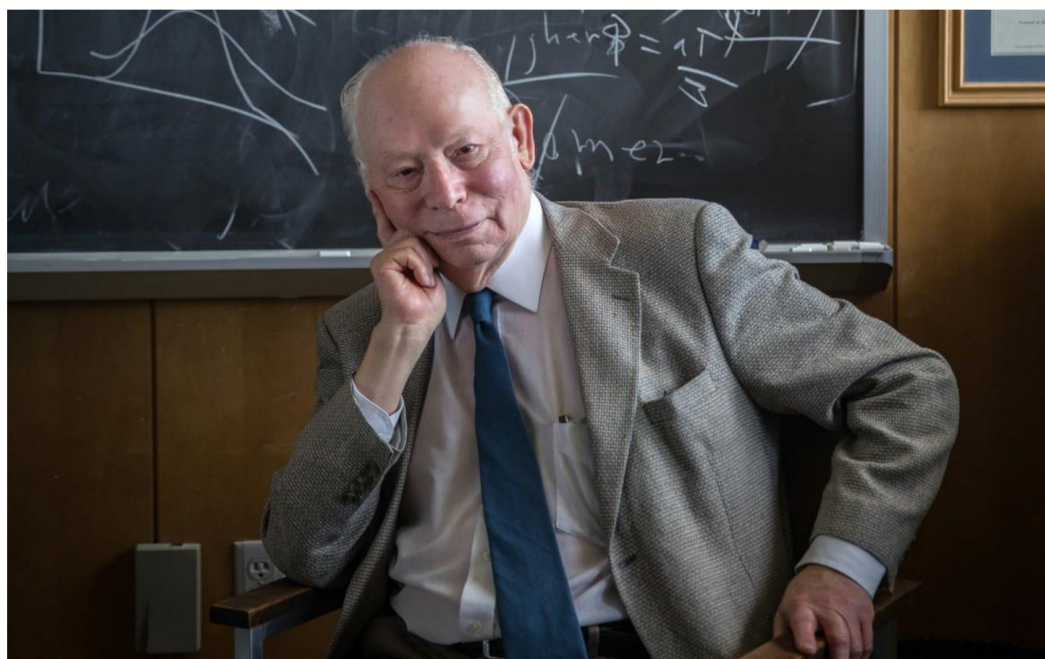
### STEVEN WEINBERG

(May 3, 1933 – July 23, 2021)

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The Nobel Prize in Physics 1979 was awarded jointly to Sheldon Lee Glashow, Abdus Salam and Steven Weinberg:

*"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current."*



Steven Weinberg at the University of Texas at Austin in Austin, Texas, August 25, 2016.

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“AS A THEORETICAL COSMOLOGIST I APPRECIATE GREATLY WEINBERG’S WORK THAT BRIDGED THE GAP BETWEEN ASTRONOMY AND HIGH ENERGY PHYSICS INSPIRING MANY TO WORK IN THIS BORDER AREA. I FOUND HIS CLASSIC TEXTBOOK “GRAVITATION AND COSMOLOGY” VERY USEFUL, AND HOPE OTHERS WILL ALSO FIND IT SO.”

: Jayant Narlikar

Jul 25, 2021

(Exclusive comment for Scientific Voyage)